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| RESEARCH ARTICLE

Linear Programming Using ABS Method

Mohammad Yasin Sorosh[1] ✉ Samaruddin Jebran[2] and Mohammad Khalid Storai[3]
[1]Ghazni Technical University, assistant professor, Department of Applied Mathematics, Ghazni, Afghanistan
[2]Kabul University, Assistant Professor, Faculty of Mathematics, Kabul, Afghanistan
[3]Kabul University, Assistant Professor, Department of Applied Mathematics, Kabul, Afghanistan
Corresponding Author: Mohammad Yasin Sorosh, **E-mail**: yasin.sorosh9@gmail.com

| ABSTRACT
Nowadays, we face many equations in everyday life, where many attempts have been made to find their solutions, and various methods have been introduced. Many complex problems often lead to the solution of systems of equations. In mathematics, linear programming problems is a technique for optimization of a linear objective function that must impose several constraints on linear inequality. Linear programming emerged as a mathematical model. In this study, we introduce the category of ABS methods to solve general linear equations. These methods have been developed by Abafi, Goin, and Speedicato, and the repetitive methods are of direct type, which implicitly includes LU decomposition, Cholesky decomposition, LX decomposition, etc. Methods are distinguished from each other by selecting parameters. First, the equations system and the methods of solving the equations system, along with their application, are examined. Introduction and history of linear programming and linear programming problems and their application were also discussed.

| KEYWORDS

ABS Algorithm, Huang Method, Implicit LU, Implicit LX, Linear Programming, Linear Inequalities.

| ARTICLE INFORMATION

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1. Introduction

Nowadays, we face many equations in everyday life, where many attempts have been made to find solutions, and various methods have been introduced. Many complex problems often lead to the solution of equations systems.

In mathematics, linear programming problems is a technique for optimization of a linear objective function that must impose several constraints on linear inequality. Informally, linear programming uses a linear mathematical model to get the best output (e.g. maximum profit, minimum work) according to given conditions (for example, only 30 hours per week, illegal work not done, etc.). More formally, in a polygon or polygon on which a function with real value is defined, the goal is to find the point in these conditions where the objective function has the most or the least value. These points may not be available, but if available, searching among the vertices of a polygon will ensure that at least one of them is found [Ahmadi, 2011].

Solving the problem by linear inequality dates back to the Fourier area. Linear programming emerged as a mathematical model and became clear during and after World War II that planning and coordination of various projects and the efficient use of scarce resources were a necessity. The US Air Force Optimized Planning Team began its serious work in June 1947. The result was the invention of the simplex method by Jorj. B. Dantzik at the end of the summer of 1947. Linear programming is interested in economists, mathematicians, statisticians, and government agencies. In the summer of 1949, a planning conference was held to plan expenditures and returns under the auspices of the Cowles Committee for Economic Research. Papers presented at this conference were compiled shortly afterwards in 1951 by T.C. Koopmans in a book entitled Production Activity Analysis and

Allocation. In the same year, Janvan Neumnn developed the theory of duality, and Leonid Khashian, a Russian mathematician, used simple techniques in pre-Dantesik economics, winning the 1975 Nobel Prize in Economics [Tofiq, 2008]. In this study, linear programming by the ABS method was investigated.

2. Solving Linear Equations Systems If F is a field, find n scalar $x_1, x_2, ..., x_n$ in the following Equation;

$$A_{11}x_1 + A_{12}x_2 + ... + A_{1n}x_n = b_1$$
$$A_{m}x_1 + A_{22}x_2 + ... + A_{2n}x_n = b_2$$
$$.$$
$$.$$
$$.$$
$$A_{n1}x_1 + A_{n2}x_2 + ... + A_{mn}x_n = b_m$$

where $b \in R^m$, $A \in R^{m\times n}$, $1 \le j \le n$ and $1 \le j \le m$. The above equation is called m equation in n unknown linear. Each n of $x_1, \ldots, x_n$ elements that hold in any of the above equations is called a system solution. This equation is briefly in the form of;

$Ax = b$

A is matrices of equation coefficients, x unknown vectors and b is the right value.

Tip 1. If the equation has at least one solution, it is consistent; and inconsistent if it does not have a solution at all [Zahedi, 2009]. First, two methods were used to solve these types of equations, which were:

2.1 Cramer's Method

One of the applications of determinants is this method, which is used to obtain the solutions of equation sets $Ax = b$. Thus, if the determinants A is opposite to the zero $(\det A \neq \circ)$ and A_k is the displacement matrix of column A with the right vector (b), then;

$$x_1 = \frac{\det A_1}{\det A}, \quad x_2 = \frac{\det A_2}{\det A}, \ ..., \quad x_n = \frac{\det A_n}{\det A}.$$

Note: The Cramer's rule is used only to solve a system of equations whose coefficient matrix is non-odd and whose order is very small. $(n \le 10)$ [Abbasi, 2011]

$(\det(A) \neq \circ)$

2.2 Inverse Matrix Method

$Ax = b$ can also be solved in this way; if there is an invertible matrix, its determinants must be zero. Therefore, we have the following assumption:

$$Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b.$$

As seen, calculating the determinants by Cramer's rule and the invertible matrix by the inverse matrix method is not easy when the coefficient matrix has large dimensions. Therefore, these methods are not widely used, and therefore mathematicians have sought newer methods that are generally divided into two categories: 1- Direct methods and 2- Repetitive methods [Jahanshahloo, 2004].

2.3 Linear Programming

We begin our discussion by formulating a specific type of mathematical programming problem. As you can see below, any linear programming problem can be designed this way. [Tehran, 1993]

A) Variables

Unknown values are the equation that must be decided.

B) Restrictions

The governing conditions of the problem are expressed as a number of mathematical equations or inequalities.

C) Objective function

It is a linear expression in which the objective of solving a problem is specified. The objective function may be Max or Min.
The general form of linear models is as follows:

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \text{) Min or Max(}$$
$$\geq b_1 \text{ or = or } s.t.\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq$$
$$\geq b_2 \text{ or = or } a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq$$
$$\geq b_m \text{ or = or } a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq$$
$$\text{unrestricted } x_1, x_2, \dots, x_n \geq 0 \text{ or } \leq 0$$

Here, $c_1x_1 + \dots + c_nx_n$ is objective functions (or standard functions) that need to be optimized and denoted by Z. Coefficients are $c_n, \dots, c_1, c_1$ coefficients of the objective function (known) and $x_n, \dots, x_2, x_1$ decision variables (variables, structural variables, or activity levels) that need to be specified. Restricted $\geq b_i$ or = or $\sum_{j=1}^{n} a_{ij}x_j \leq$ indicates the first constraint (implicit or functional, structural or technical), and the a_{ij} coefficients are called technical coefficients.

The column vector whose first component is b_i is called the right vector. A set of $x_n, \dots, x_1$ variables that holds in all restricts called feasible region.

The LP problem can be summarized as follows:

$$(Max \quad Min) z = \sum_{j=1}^{n} c_j x_j$$
$$s.t. \qquad \sum_{j=1}^{n} a_{ij} x_j \leq \quad = \quad b_i \qquad i = 1,000,m \qquad (1\ .1)$$
$$x_j \geq 0 \quad \leq 0 \qquad j = 1,000,n$$

The matrix shape (1. 1) is as follows:

$$(Max \quad Min) z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$
$$s.t. \qquad a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq \quad = \quad \geq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq \quad = \quad \geq b_2$$
$$.$$
$$.$$
$$.$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq \quad = \quad \geq b_m$$
$$x_1, x_2, \dots, x_n \geq 0 \quad \leq 0$$

Where, $C = (c_1, c_2, \dots, c_n)$ is objective function coefficients, $x^t = (x_1, x_2, \dots, x_n)^t$ the vector of variables, $A = \begin{bmatrix} a_{11} & a_{12} & \dots a_{1n} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$ is restricted coefficient matrix (technical coefficients), and $b^t = (b_1, b_2, \dots, b_m)^t$ is the requirements vector.

In economic problems, maximization and minimization problems are often seen in a special way, which we call the focal (conventional) form. The simplex method also specially solves the problem, which we call the standard form. Table (1) shows both focal and standard forms of maximization and minimization problems [Esmaeili, 2001].

Table 1- Focal Shape and Standard of Maximization and Minimization

	Maximization	Minimization
Focal (Conventional)	$Max\ Z = Cx$ $s.t.\ Ax \le b$ $x \ge \circ$	$Min\ Z = Cx$ $s.t.\ Ax \ge b$ $x \ge \circ$
Standard	$Max\ Z = Cx$ $s.t.\ Ax = b$ $x \ge \circ$	$Min\ Z = Cx$ $s.t.\ Ax = b$ $x \ge \circ$

3. Linear Programming Hypotheses

To illustrate an optimization problem in linear programming, several mandatory assumptions are needed in the problem formulation discussed earlier [Spedicato, 2003].

3.1 Proportionality Assumption

The proportionality assumption means that more use of activity j costs more; i.e., there are no savings or discounts or savings and no start-up costs to start the activity.

3.2 Additivity Assumption

The additivity assumption ensures that the total cost is the sum of the individual costs and the total share or restriction i is the total share of the individual activities. In other words, there are no substitution or interaction effects between activities. [Spedicato, 1997].

3.3 Divisibility Assumption

The divisibility assumption states that decision variables can be divided as much as necessary and are, therefore, allowed to take incorrect values.

3.4 Certainty Assumption

The coefficients of b_i, a_{ij}, c_j are certain, and according to the certainty Assumption, no probable or accidental element is inherently present in demand, cost, prices, existing industries, applications, etc.; these coefficients should be approximated with their equivalent if there is any probable or accidental element [Zhang, 1998].

4. Problem Manipulation

With a series of conversions, any LP problem can be converted to any other form of LP problem; even some non-linear problems can be turned into an LP problem.

4.1 Convert the Objective Function from Maximum to Minimum and Vice Versa

Assumption: If we want to minimize the objective function $Z = c_1x_1 + \ldots + c_nx_n$ Max, it is enough to multiply all the coefficients of the objective function by negative. In this case, the solution method will change to Min $-Z = c_1x_1 - c_2x_2 \ldots - c_nx_n$, but note that to equalize the two objective functions at the end of solving the minimization problem, we must multiply the value of the minimization objective function by negative as follows:

$$Max\ c_1x_1 + \ldots c_nx_n = -Min - c_1x_1 - \ldots c_nx_n$$

Note that the nature of the coefficient of two negatives is different; the first negative changes the method, and the second negative equals the value of the two functions.

4.2 Change the Inequality Direction

In LP problems, we allow the unequal sides to be multiplied by the negative to change the inequality direction. For example:

$$px_1 + qx_2 \le b \Rightarrow -px_1 - qx_2 \ge -b$$

4.3 Inequalities and Equalities

To convert equality to inequality, equality can be written as two inequalities:

$$px_1 + qx_2 = b \Rightarrow \begin{cases} px_1 + qx_2 \le b \\ px_1 + qx_2 \ge b \end{cases}$$

And to convert inequality to equality, you can do the following:

$$\begin{array}{ll} px_1 + qx_2 \ge b & \Rightarrow px_1 + qx_2 - S = b\ ,\ S \ge 0 \\ p'x_1 + q'x_2 \le b' & \Rightarrow p'x_1 + q'x_2 + S' = b' \quad S' \ge 0 \end{array}$$

Note that $S, S' \ge \circ$ are generally called auxiliary variables.

4.4 Convert Absolute Value of Restrictions

The following restrictions can be divided into two linear restrictions:

$$|px_1 + qx_2| \le b \Rightarrow -b \le px_1 + qx_2 \le b \quad \Rightarrow \begin{cases} px_1 + qx_2 \le b \\ px_1 + qx_2 \ge -b \end{cases}$$

Note that the following restriction cannot be construed as an LP model, and these restrictions will be discussed in proper planning.

$$|px_1 + qx_2| \ge b \Rightarrow \begin{cases} px_1 + qx_2 \ge b \\ \quad or \\ px_1 + qx_2 \le -b \end{cases}$$

Because only one of these two must always be present, the problem is not an LP problem.

4.5 Unrestricted Variables

Method 1) Suppose variable x_j is an unrestricted variable (free in sign); it can be replaced by two unrestricted variables as follows:

$$x_j = x'_j - x''_j \qquad x'_j, x''_j \ge 0$$

Besides, if x_j unrestricted with absolute value appears in the objective function, then $|x_j|$ can be substituted as follows with two restricted variables:

$$|x_j| = x'_j + x''_j \qquad x'_j, x''_j \geq 0$$

Method 2) If in the first method, we have k unrestricted variables in the problem, we can replace x''_j variables $x'' = Max\ \{x''_j : x''_j \geq \circ\}$ instead of all the variables:

$$x_j = x^+{}_j - x''_j \qquad x^+_j \geq 0\ ,\ j = 1, \ldots, k$$

Method 3) If $x_j \geq l_j$ can be written as $x_j - l_j$ and by changing the variable $x'_j = x_j - l_j$ so that it is $x'_j \geq \circ$, the variables can be restricted, and if $x_j \leq u_j$ it can be written in the same way as $u_j - x_j \geq \circ$ and by changing the variable $x'_j = u_j - x_j$ so that it is $x'_j \geq \circ$, the variables would be restricted.

Method 4) If $|x_j| \leq u_j$ then it can be written as $x_j \leq u_j, x_j \geq -u_j$ and using the third method mentioned, the variables can be restricted. But if it is $|x_j| \geq l_j$, the problem will not be simply the above, and this will be discussed in integer programming, and with a variable of zero and one, the problem can be solved.

4.6 Maximin and Minimax Objective Function

Consider the $Min\ (Max\{2x_1 + 4x_2, 3x_1 - x_2\})$ objective function. This objective function is a composite objective function, and the objective function of LP problems has only Max or Min. This problem can be solved with a simple variable change.

Suppose $y = Max\{2x_1 + 4x_2, 3x_1 - x_2\}$, then we have:

$$\begin{aligned} &\min y \\ &s \cdot t \quad 2x_1 + 4x_2 \leq y \\ &\qquad\ \ 3x_1 - x_2 \leq y \end{aligned}$$

4.7 Geometric Solution

Consider the problem $\begin{cases} \min\ \ z = cx \\ s\ .t\ \ \ Ax \geq b \\ x \geq \circ \end{cases}$, every x that holds in $Ax \geq b$ inequalities, it is the solution of the problem, and if it holds in condition $x \geq \circ$, we call a feasible solution to the problem.

The set S is defined as follows:

$$S = \{x | Ax \geq b, x \geq \circ\}$$

is a linear programming problem in the set of feasible or non-feasible solutions.

4.8 The Optimal Solution

We call $x^{*} \in S$ the optimal Solution $S \neq \phi$ if:

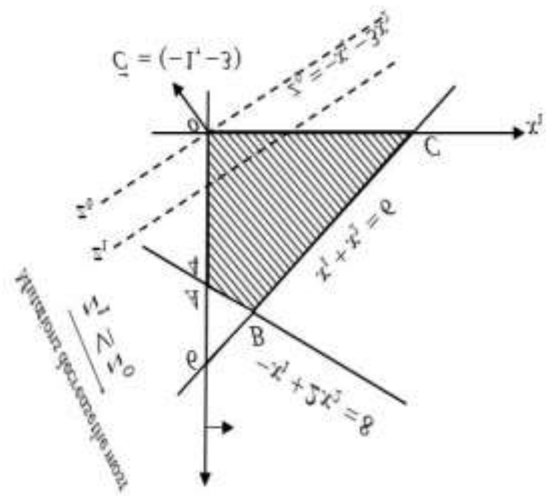
$$\forall x (x \in S \Rightarrow Cx^{*} \leq Cx)$$

We now examine the concepts mentioned in this section with an example of two decision variables.

Example

Consider the following problem:

$$\begin{aligned} &Min \quad z = -x_1 - 3x_2 \\ &s \cdot t \quad\quad x_1 + x_2 \leq 6 \\ &\quad\quad -x_1 + 2x_2 \leq 8 \\ &\quad\quad\quad x_1, x_2 \geq 0 \end{aligned}$$



To solve the linear programming problem, we first draw all the constraints equally, and then, by examining an arbitrary point from the two regions created by this line, we determine the desired region that its solution is unequal, sharing all the solutions of the inequalities is the feasible region of the problem.

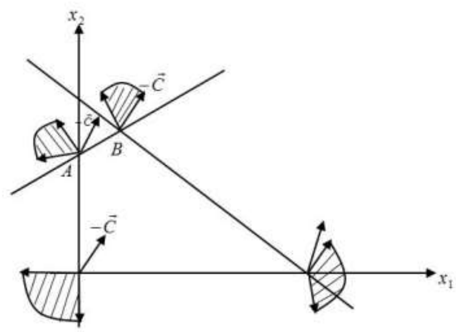
Note that the polygon OABC shares all four quadratic solutions $x_1 + x_2 \leq 6$, $x_2 \geq \circ, x_1 \geq \circ, -x_1 + 2x_2 \leq 8$. It is observed that $S \neq \phi$ is the feasible region of the problem, which is obtained by sharing all the inequalities of the problem.

Method 1) To obtain the optimal solution, we first draw the line $z = \circ = -x - 3x$ and then move the objective function parallel to itself and in the direction of maximum reduction $(-\vec{C})Z$, the last point that the line Z will have with S will be the optimal point of the problem in which Problem $B(\frac{4}{3}, \frac{14}{3})$ with the value of the objective function will be $Z^{*} = \frac{-46}{3}$.

Method 2) S has 4 vertices.

First, we draw the vectors perpendicular to the constraints passing through the vertex points to the outside of the area to be zoned, and we hatch the resulting cone. In the problem of minimization $-\vec{C}$ (in the problem of maximization we place $(\vec{C}$ parallel and in the same direction on all vertex points, the vertex that $-\vec{C}$ (at max. $\vec{C}$) falls inside the cone is the optimal vertex for the corresponding problem. As can be seen, the point $B(\frac{4}{3}, \frac{14}{3})$ is the optimal point of the minimization problem (Figure 1).

Figure 1- Optimal minimization problem



In the geometric method and drawing the shape, we may encounter 4 modes to find the optimal solution with each of the expressed methods:

A) The finite unique optimal solution: In this case, there is only one optimal point for the problem.

Note 1. In the case of a finite unique optimal solution, the set of optimal points is a single member.

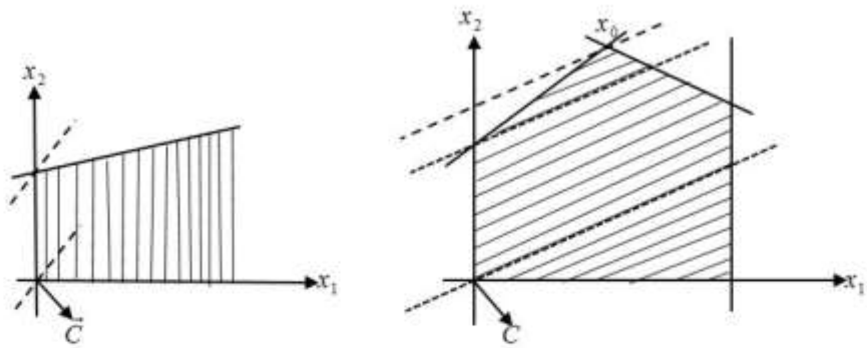


Figure 2- Finite Optimal Solution

Note 2. As shown in Figure (2) (b), the justified area can be unlimited, but the optimal solution is limited.

B) Infinite solution: In this case, the solution is infinite.

According to Figure (2), it can be seen that the movement of the objective function in the opposite direction of $\vec{C}$ to minimize the objective function, the objective function never leaves the area, so the value of the objective function will reach $-\infty$ miles and the solution to the minimization problem will be infinite.

Note 3: The necessary condition for the optimal infinite solution is an infinite justified area.

Note 2: In the case of the infinite optimal solution, the set of optimal points is empty.

Note 5: In practical problems, if the problems find infinite optimal solutions, the problem modeling must have been done incorrectly.

C) Finite optimal solution: In this case, if the optimal objective function contains more than one optimal point, we will have finite multiple optimal solutions;

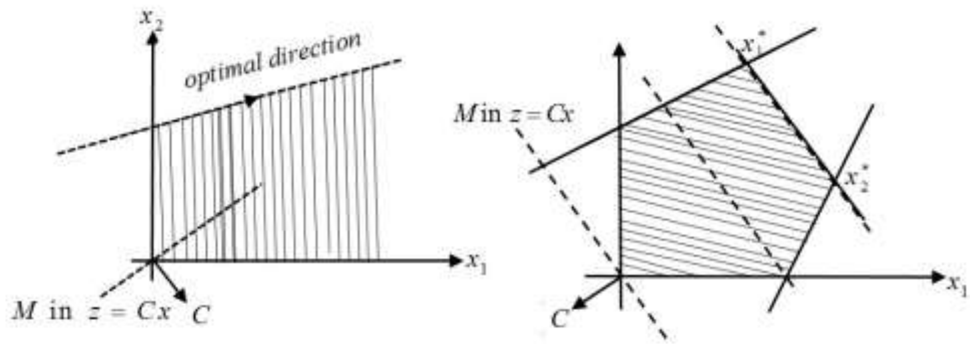


Figure 3 - The Finite Multiple Optimal Solution

As can be seen from Figures (3) (a) and (b), the problem has more than one optimal point (infinitely optimal point).

Note 6: In the case of finite multiple optimal solutions, the set of optimal points is infinite points. (If the problem has more than one optimal point, it has an infinite optimal point).

Note 7: In two-dimensional space, the necessary condition for the multiple optimal solutions is the parallelism of the objective function with one of the constraints.

D) Impossible (unjustified): In this case, there is no commonality between all the constraints of the problem.

4.9 Requirement Space

The problem of linear programming can be solved and interpreted geometrically in another space called requirement space.

Consider the following equation:

$$\begin{aligned} &M\text{in} && Z = Cx \\ &s \cdot t && Ax = b \\ & && x \geq 0 \end{aligned}$$

We rewrite the problem as follows:

$$\begin{aligned} &M\text{in} && Z = Cx \\ &s \cdot t && \sum_{j=1}^{n} a_j x_j = b \\ & && x_j \geq 0 \qquad j = 1, \ldots, n \end{aligned}$$

The vectors are a_n،،a،a_2 $_1$rows A, and we want to find the negative $x_n, \ldots, x_1$ so that $Z = \sum_{j=1}^{n} c_j x_j$, $\sum_{j=1}^{n} a_j x_j = b$ is minimized.

Note that the set of vectors $\sum_{j=1}^{n} a_j x_j$ in which there are $x_1, x_2, \ldots, x_n \geq \circ$ cones created by the $a_n, \ldots a_2, a_1$ vectors is, therefore, problematic if the vector b is placed in this cone.

4.10 ABS solution to a specific linear programming problem

We now consider the problem of linear programming, which $rankA = m, m \le n, b \in R^m, x \in R^n$

$$\max c^T x : Ax \le b \qquad (1)$$

And using the above problem is equivalent to:

$$\min(c^T M \gamma + C^T H^T q): \quad \gamma \in R^m, q \in R^n \qquad (2)$$

We have,

$$\bar{c} = M^T c$$

$$I^+ = \left\{ i \mid \bar{c}_i > \circ \right\},$$

$$I^\circ = \left\{ i \mid \bar{c}_i = \circ \right\},$$

$$I^- \left\{ i \mid \bar{c}_i < \circ \right\}$$

4.11 Theorem

Suppose we use the ABS algorithm for A as in the proposition of 308, then we have $x^* = A_{W_m}^{-T} b$:

(a) If $Hc \neq 0$ then problem (1) is infinite and has no solution.

(b) If $Hc = 0$ and $I^- \neq \phi$ then problem (1) is infinite and has no solution.

(C) If $Hc = 0$ and $I^- = \phi$ then an infinite number of optimal solutions for (12. 3) form

$$x = x^* - H^T q - \sum_{j \in I^\circ} \gamma_j M e_j$$

We have,

$$q \in R^n \text{ و } \gamma_j \ge \circ \;\; j \in I^O \text{،}$$

Are arbitrary and ej is the unit vector of j in R^m.

4.12 Proof:

(a) Given (2), Hc vectors are not zero, q can be chosen appropriately to obtain the desired small value for the objective function.

We now assume that $Hc = 0$ using the above symbol, such as Problem (1), can be written as follows:

$$\min z = \sum_{j \in I^-} \bar{c}_j \gamma_j + \sum_{j \in I^+} \bar{c}_j \gamma_j + \sum_{j \in I^\circ} o \gamma_j : \gamma_j \ge \circ \;\; \forall_j$$

(b) Since $I^- \neq \phi$ after $k \in I^-$ is available. By placing $\gamma = tek$ then, we have; $Z \to -\infty$ when $t \to \infty$ is therefore infinite and has no solution.

(c) In this case $z = \sum_{j \in I^{+}} \bar{c}_j \gamma_j + \sum_{j \in I^{\circ}} \circ \gamma_j$, where the z-minimizers are written as $\gamma = \sum_{j = I^{\circ}} \gamma_j e_j$, in which $\gamma_j \geq \circ, j \in I^{\circ}$ are arbitrary, so the optimal solutions in (1) are $x = x^{*} - H^{T} q - \sum_{j \in I^{\circ}} \gamma_i M e_j$, in which $q \in R^{n}$, $r_j \geq 0$, and $j \in I^{\circ}$ are arbitrary.

Note 11. 3: According to the characteristics of ABS, $HA^{T} = 0$ and hence H nullity=A^T is the condition for $Hc = \circ$. Regarding Koohan Taker condition $c = A^{T} u$, for u, since A^T has a full column rank, then u is unique. The $\frac{1}{d}\bar{c}$ vector, on the other hand, holds $A^{T} u = c$ because lines $A_{w_m}^{-1}$ are linearly independent, and the solution $A^{T} u = c$ is equivalent to the solution $A_{w_m}^{-1} A^{T} u = A_{w_m}^{-1} c$.

Since $A_{w_m}^{-1} A^{T} = I$ then $u = A_{w_m}^{-1} c = \frac{1}{d}\bar{c}$. So when w$_i$ are selected, so that $\bar{c}, d^{m} > 0$ has the same symbol. Just as multiplications (Lagrange) are components. If $\bar{c} \geq 0$ then the problem is infinite and has no solution, then we find that for $i \in I^{-}$, $u_i < 0$ and for every $i \notin I^{-}$, $u_i \geq 0$. If $\bar{c} \geq \circ$ then, there are an infinite number of optimal solutions to the problem.

Here, as expected for optimization, for each i, $u_i \geq 0$. Using the above results, the following algorithm is proposed to solve the linear programming problem (1):

1) We use the ABS algorithm with matrix A.

$A_{w_m}^{-T}$ and H$_1$

$$d = \left|\det(w_m^{T} H_1 A^{T})\right|$$
$$M = d A_{w_m}^{-T}$$

Calculate.

2) If $Hc \neq 0$ then stop (the problem is infinite so there is no solution).

3) Hypothesis $\bar{c} = M^{T} c$ and its form are in the form of the following sets:

$$I^{+} = \left\{ i \mid \bar{c}_i > \circ \right\}$$
$$I^{-} = \left\{ i \mid \bar{c}_i < \circ \right\}$$
$$I^{\circ} = \left\{ i \mid \bar{c}_i = \circ \right\}$$

4) If $I^{-} \neq \phi$ then stop (it is an infinite problem and has no solution).

5) If

$$x^{*} = A_{w_m}^{-T} b \ ,(I^{-} = \phi)$$

Calculate.

Which are the optimal solutions

$$x = x^{*} - H^{T}q - \sum_{j \in I^{\circ}} \gamma_j M e_j$$

Where,

$$q \in R^{n}, \gamma_j \geq 0, j \in I^{O}$$

And are arbitrary numbers, and ej denotes the unit vector j in R^m . Stop.

We have used ABS algorithms to solve real linear equations for full-rank linear inequalities and linear programming problems in which the number of inequalities is less than or equal to the number of variables, and the optimal and infinity conditions in the algorithm are obtained.

5. Conclusion

In this study, the ABS method is proposed to solve various equations and linear inequalities. This algorithm is a well-defined algorithm when the rank of the matrix A (coefficient matrix) is equal to m. In this case, the method is more efficient, and the number of calculations is less.

These methods have the ability to produce suitable solutions for inactive equations. They are also effective in solving large equations. They are more efficient at solving large equations (large m or n) than conventional direct methods. The ABS method is also used to solve linear programming problems. This method is equivalent to the Symplex method with the Bland rule in linear programming to find a feasible region that can be found from $Ax \geq b$ linear inequality equations.

$$A \in R^{m \times n}, b \in R^{m}, x \in R^{n}, m \leq n$$

Which is used in a finite number of steps, and the results obtained for ABS methods are not only theoretically important but also remarkable from the point of view of numerical calculations.

5.1 Research Suggestions

Here are some of the works that can be done in future studies:

1- Applying the ABS method to solve the system of nonlinear equations with the incomplete rank

2- Applying the ABS method to solve the system of nonlinear inequalities with the incomplete rank

3- Applying the ABS method to solve the system of nonlinear equations with full rank

4- Applying the ABS method to solve integer mixed planning problems

5- Applying the ABS method to solve integer planning problems

6- Applying the ABS method to solve quadratic planning problems

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