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Using Mathematics Modelling to Teach Mathematics Word Problems

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ABSTRACT

The shift of curriculum from non-contextualized to contextualized mathematics has called schools to focus more on competency based items. The real world word problem has made teaching and learning mathematics more open and less predictable hindering the performances of students. Therefore, the intervention research was conducted with 51 students of class VI of Choekhorling Lower Secondary School, Sarpang, Bhutan to address the difficulties in solving word problems in mathematics. The mathematical modelling was used as an intervention for a period of 10 weeks to narrow the gap between mathematical and real world problems. The study concluded that comprehending the problem and translating real world problem to mathematical problem were the main causes of difficulties in solving competency based items by the participants. Moreover, there was mean difference in pre-intervention and post intervention scores ($p<0.001$) which indicated statistically significant to ascertain that mathematical modeling can help students' mathematical leaning become more meaningful and teachers can obtain the patterns of students' problem-solving competence.

1. Introduction

Word problem solving in mathematics is an imperative aspect of learning mathematics and mathematical thinking (Vula & Kurshumlia, 2015). The shift of Bhutanese mathematics from systematic mathematics to understanding mathematics in the beginning of 21st century has demanded more mathematical thinking. Such kind of reform is to help students construct knowledge by creating learning conditions in which students extend their personal knowledge to include theoretical knowledge (Palha, Dekker & Gravemeijer, 2015). The mathematics workbooks and textbooks contain more of contextualized problems than non-contextualized problems. Even in the examinations like the Mathematics Competency Based Assessments for class VI board examination conducted by Bhutan Council of School Examination and Assessment (BCSEA) consisted 50% conventional items and 50% competency based items. Since few years, the Mathematics items were contextualized to different situations and more emphasized were on processes and procedural skills than declarative knowledge (BCSEA, 2018). Contextualizing the mathematics problems has challenged our teachers and students with difficulties in solving word problems.

This study tried to uncover the underlying problems of having difficulty in teaching and solving the word problems in mathematics classes by employing a profound mathematical model (see intervention) using the qualitative research tools of observation and interviews. The findings from this study can be a means to make word problems meaningful and draw a link between real world problem and mathematical problems.

1.1 Problem Statement

Students cannot perform well in the examination when it comes to word problem-solving questions and it is an area of difficulty and frustration for a considerable number of students (Osman, et al., 2018). The student performance in the CBA test conducted by BCSEA has surprised many educators and learners in 2017. BCSEA (2018) reported that class VI students scored mean score of 57.96% in Dzongkha (Bhutan's national language), 54.33% in English, 35.33%in Mathematics, 52.31% in Science and 55.83% in Social Studies in National level CBA test. Alarmingly, mean score and the range of mathematics score was as low as 30.75% to 43.52% which depicts poor performances of the students of class VI across all the schools in the nation in mathematics.

The performances of class VI students in Choekhorling Lower Secondary School was not different from the national result. Word problem is the main cause for the students to score low marks in mathematics. Moreover, our students have failed to apply non-contextual solution to the contextual problem as they are never taught. The mathematical concepts (non-contextual) learnt in the class appears alien to the address the real life problems (Contextual problem).

Therefore, we used a mathematical model to help our students solve the word problems with understanding rather than using the traditional strategy by asking our students to look for key words and clue in the question. Jonassen (2003) found that "the search for key words strategy was common in classrooms where students had little problem solving success, and that this strategy was taught and re-taught to struggling math students' year after year in spite of their lack of success" (as cited in Pearce, Bruun, Skinner & Lopez-Mohler, 2013, p. 12). Class VIA &B students of Choekhorling Lower Secondary School participated in this study to learn how to solve word problems meaningfully through mathematical modelling.

1.2 Research Questions

1.2.1 Overarching Question

Can teaching of mathematical model help learners solve competency based problems?

1.2.2 Guiding Questions

1. What are the causes of difficulties in solving word problems ?
2. What is the impact of mathematical model on students' abilities to solve word problems?
3. How can mathematical model help learners make link between non-contextualized and contextualized problems?

3. Literature Review

A word problem is combination of numbers and words designed to help students apply mathematics concepts to real life situations (Salemeh & Etchells, 2016). It is a contextualized problem (Pearce, et al., 2013). Word problems provide opportunities for learners to use mathematical tools to promote link between mathematics and real life context (Angateeah, 2017) as the problem instruction tells a story or describes verbally a situation (Thevenot, Devidal, Barrouillet & Fayol, 2007). Most of the word problems are competency based and contextualized to real life which may need a different approach from the conventional based or non-contextual problems which sought facts and figures. In literatures, the word problems are also referred as "verbal arithmetic problems, story problems and problem solving situations" (Pearce, et al., 2013, p.3).

The lack of linguistic knowledge is a source of difficulties with word problems. It is argued that the competency based items are becoming increasingly "a test of a student's ability to read and understand the problems instead of a test of computation skills" (Pearce, et al., 2013, p.3). Moreover, the written tests are predominant forms of assessment in mathematics teaching (Bergqvist & Osterholm, 2010) placing the problem comprehension process at the core of finding solutions of arithmetic word problems (Hegarty, Mayer & Monk, 1995).

The success of a student in solving word problem depends on the process of comprehending the problem. The thinking processes used by individuals or small groups as they solve problem are the focus of recent researchers (Angateeah, 2017). Angateeah (2017) studied the cognitive processes undergone by students who have difficulties in solving word problems by using the framework proposed by Montague (2003, in Angateeah, 2017). 15 students who were interviewed for the study depicted that low achievers face difficulties in decoding the language in the problem. Moreover, carelessness in reading the question or in calculation was found to lead to wrong answer followed by miscomprehension and poor procedural skills. Hence, the author suggested that the teaching process should lay emphasis on reading and interpretation of problem, encourage to use various representations and strategies to solve problem and urge students to develop strategies to check their answer. This study, therefore, employed a model to solve word problem to bridge the gap stated by the author.

In another study, Hegarty, Mayer and Monk (1995) examined 38 undergraduates who were classified into successful and unsuccessful problem solvers depending on the errors (0 or 1 error as successful and more than 3 as unsuccessful) they made in solving four sets of 48 arithmetic problems. They concluded that unsuccessful problem solvers base their solution plan on the numbers and keywords which authors call *direct-translation strategy* and the successful students the *problem model strategy* where they try to construct a mental model based on the situation being described in the problem. Moreover, it is learnt that there is no evidence of practice causing unsuccessful problem solvers to change towards a problem-model strategy nor successful problem solvers to change towards a direct-translation strategy. Unsuccessful problem solver did not change to a problem-model strategy as it appeared that they did not develop this strategy during the years of mathematics instruction.

For this study, we used mathematical modelling to address the problems with solving word problems. Modelling is "the entire process leading from the original real problem situation to a mathematical model (Mousoulides, 2007, p. 11)". It is a problem solving process which includes a rich cognitive and metacognitive activities that facilitates the understanding of the relations within the nature of the problems from real world, the assumptions and the relations among assumptions (Hıdıroğlu, Dede, Ünver &Güzel, 2017).

The teachers and students face difficulties in mathematics problem solving due to incompetency in acquiring required mathematics skills and lack in cognitive abilities of learning (Tambychik & Meerah, 2010). Hence, Blum and Ferri asserts that mathematical modellings are usually designed to:

a. Help learners to better understand the world,
b. Support mathematics learning (motivation, concept formation, comprehension, retaining)
c. Contribute to develop various mathematical competencies and appropriate attitudes,
d. Contribute to an adequate picture of mathematics (Blum & Ferri, 2009).

Hence, we have chosen mathematical modelling as it has many benefits in teaching students how to solve word or contextualized or competency based items.

4. Methodology

4.1 Data Plan

There are various methods to collect information in qualitative research. Marshall and Rossman, (2011) states that the four primary methods that qualitative researches rely on gathering information are (1) participating in the setting, (2) observing directly, (3) interviewing in depth, and (4) analyzing documents and material culture, with varying emphases (as cited in Wangdi, 2016). For this study, the researchers will do participatory study as involvement is the core aim of action research. Moreover, observation and interviews were used to collect data at different stages of the study as shown in the following figure.

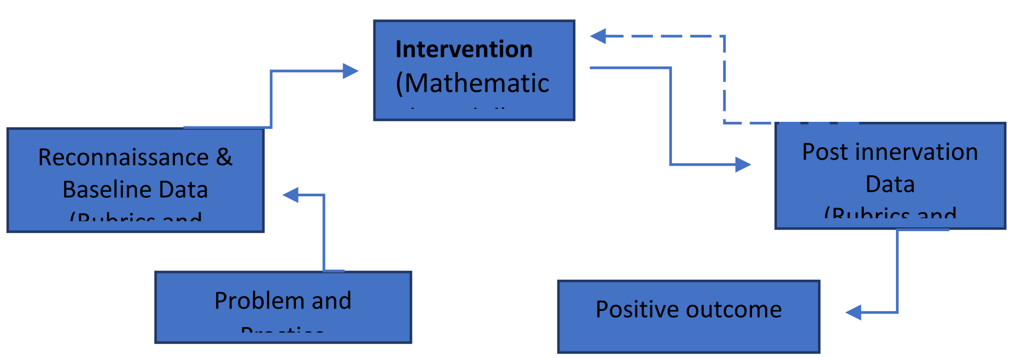


Figure 1 Data collection plan model

The baseline data was collected using two questions from the CBA test conducted by BCSEA since 2014 till 2018 which were randomly selected. A 20 minute- test was administered and students' works were assessed by two mathematics teachers using the rubrics to avoid researcher bias.

After 10 weeks of intervention period, post intervention data collection was carried out which is known as the post intervention data. The same 20 minute- test was conducted and followed similar procedures in assessing the students' performances by two mathematics teachers. Students were not informed about the same test item that was administered.

Finally, the baseline data and post intervention data were compared to authenticate the significance of the mathematical modelling.

4.2 Intervention Plan: Mathematical modelling to solve word problem in mathematics

We taught mathematical model to help students solve word problems more confidently for the period of 10 weeks (August second week till October end, 2019). All the contextualized problems were solved using the model and followed by student's self-reflection and evaluated the progress using the rubrics prepared inline to the mathematical model. SEE ANEXURE 2.

There are many models proposed by different researchers (Table 1).
<u>Table 1: mathematics models</u>

Krulick & Rudnick (1996 in Tambychik & Meerah, 2010)	Tambychik & Meerah, (2010)	Blum & Leiß (2007, in Blum & Ferri, 2009)	Blum & Ferri (2009)	Singer & Voica (2013)
Reading and thinking; Analyzing and planning; Organizing strategy Getting the answer; Confirmation of the answer	Reading and understanding problem Organizing strategy and solving problem Confirmation of the answer and process	Constructing Simplifying Structuring Mathematizing Working mathematically Interpreting Validating Exposing	Understanding task Establishing model Using mathematics Explaining result	Decoding Representing Processing implementing

The following model adopted from previous researchers (Blum & Ferri, 2009; Krulick & Rudnick, 1996; Singer & Voica, 2013; Tambychik & Meerah, 2010) was used for this study so that students of class VI can understand the model. It is guided by the aforementioned purposes of mathematical modelling which was proclaimed by Blum and Ferri 2009). The solid arrows show the steps of solving the word problem and the dotted arrow shows the repetition of the steps if students couldn't reach to the desired outcome.

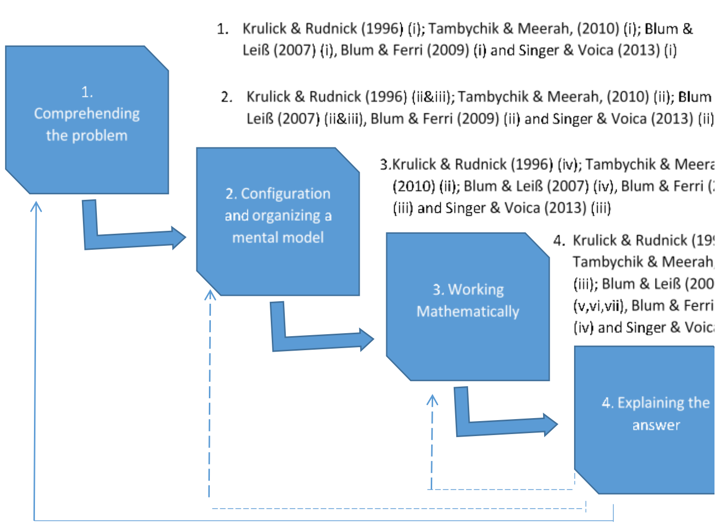


Figure 2 Mathematical model

There are four basic steps in this model namely comprehending the problem, configuration and organizing a mental model, working mathematically and explaining the answer. The solid arrow represents the successful completion of

each step and dotted arrow represents the revisiting of the certain steps in case of undesirable answers. The first step of this model requires students to demonstrate understanding of the problem comprehensively. The students were asked to rephrase the question in their own language at this phase during the intervention activities.

The second phase known as configuration and organizing a mental model requires students to draw a link between real world problem and the mathematical problem by drawing familiar diagrams or tables. The real world objects are organized using the mathematical shapes and diagrams.

The third phase requires students to use the mathematical formula or procedures to solve the mathematical problem students have linked from real world problem in phase 2. Working mathematically is a process of translating a real-world problem into mathematical problem by representing mathematical diagrams and tables.
Finally, the fourth step demands students to answer the real world problem by using the answers students got solving the problem mathematically.

The worksheet based on this model which can be seen in ANNEXURE 3 was used until the students to get accustomed to all the phases of this model. The last two weeks of intervention period devoted in solving the problem without using the worksheet but incorporating all the phases.

5. Results and Discussion
5.1 *Result*

The findings from the study conducted with class six students (N=51, male=27 and female=24) and two mathematics teachers (male=2) for a period of 10 week- intervention period is presented by reporting the response from students followed by comparison of baseline and post intervention data to see the overall intervention plan gain.

The sample of CBAT questions and the sample of desired student work is shown hereunder.

Question: *Maya wants to make two roofs with same area as shown below. One is tall and narrow and other is short and wide. Why is that possible? Explain using example.*

Roof A *Roof B*

Figure 3: CBAT sample adopted from BCSEA, 2016, Section B, Question 5(b))

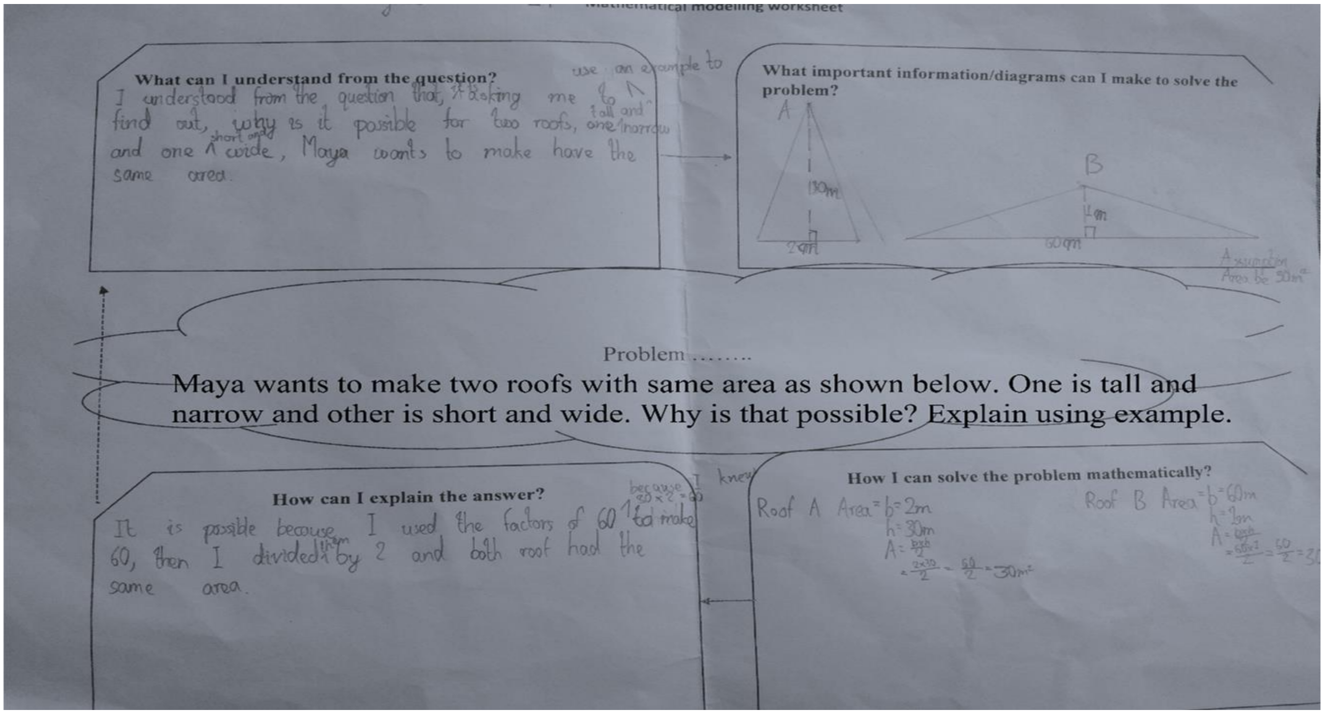


Figure 4 sample response

It was observed that students had difficulty in comprehending the questions. Most of the students could understand the questions partially or they could focus only on one or two key information. This has created difficulty in making link to the other phases of modelling process.

Question: *Maya wants to make two roofs with same area as shown below. One is tall and narrow and other is short and wide. Why is that possible? Explain using example.*

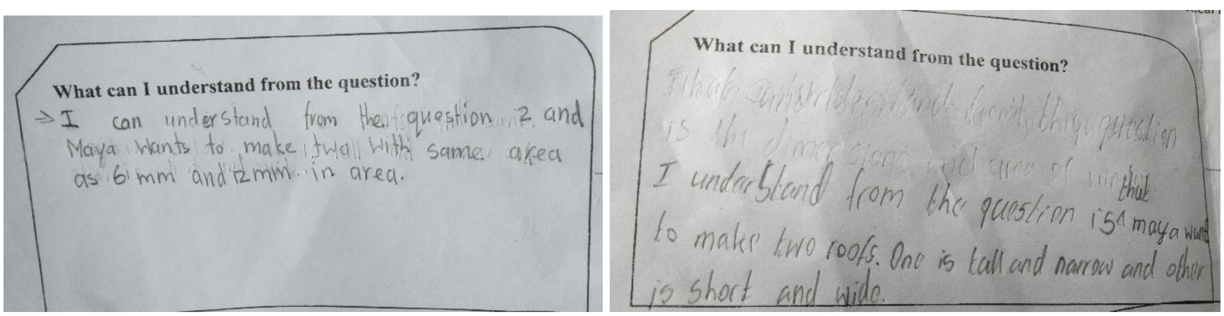
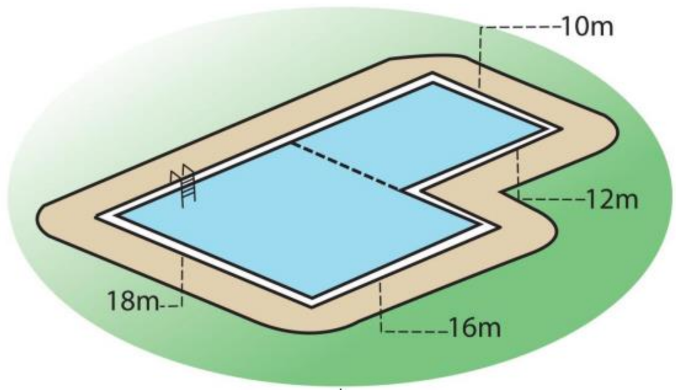


Figure 5 Partial understanding of the question- S51 and S26

In the above example, the first response clearly indicates that the student couldn't understand the problem and second shows that the child could barely understand the actual demand of the question.

Students had to translate the real world problem to familiar conventional problem or link to mathematical problem which was addressed in phase two of modelling process. It was observed that few students couldn't draw any relationship between the real world problems to mathematical problem. Some of the participants reported that they were never taught how to translate a real world problem to mathematical problem. This has led to students' lack of success in solving CBAT problems.

Some of the examples of translating a problem of swimming pool to a compound shape is shown below:



Find the perimeter and area of the surface of the swimming pool.

(Figure 6. Singapore Mathematics Assesssment Pedagogy Project, p. 67)

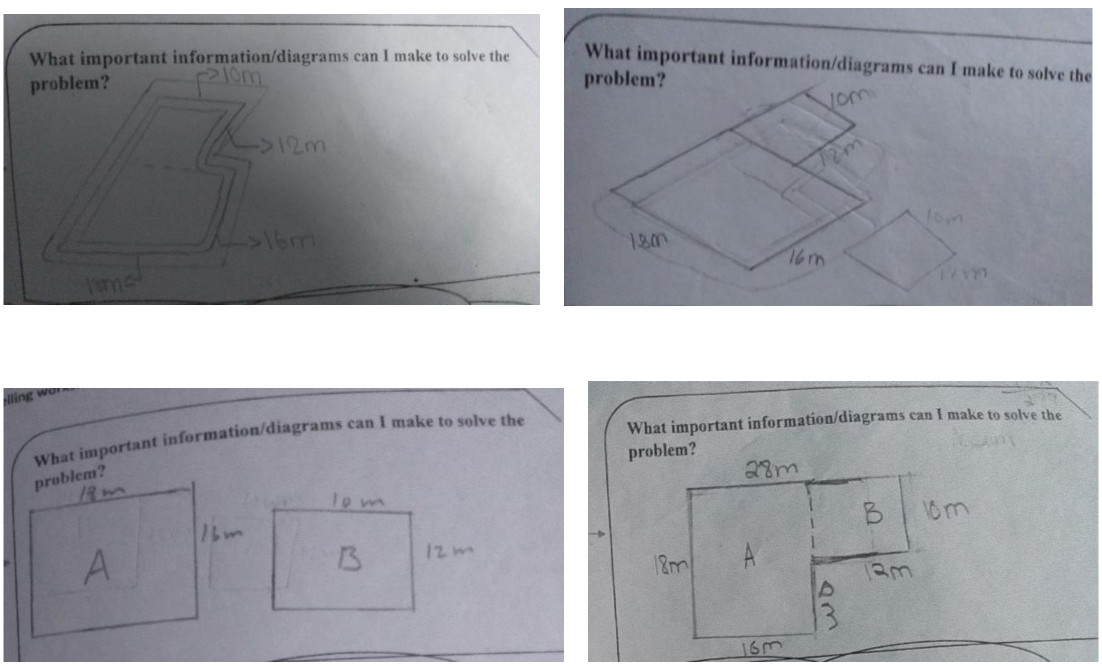


Figure 7: Sample response of translating real world to mathematics problem

The success of translating a real world problem to mathematical problem have contributed to the overall success of solving the competency based problem.

The success of third phase of mathematical modelling called working mathematically strongly depend on the success of former phases. Students who couldn't comprehend the competency based problem and translate the problem to routine problem were observed failing to apply formula or other problem solving strategy even if they know it.

Moreover, it was observed that students who could complete the phases till working mathematically seldom forgot what they are answering. They fail to answer the actual question and stops once they get mathematical answer.

The baseline and post intervention mean comparison was done by conducting paired sample T-test using SPSS to see the intervention gain.

Table 2: Paired Samples T-test (Question 1)

	Pre-intervention		Post intervention		Paired Differences			
	Mean	Std. Deviation	Mean	Std. Deviation	Mean	Std. Deviation	df	Sig. (2-tailed)
1. Comprehending the problem	1.11	.27	2.76	.67	1.65	.61	50	.000
2. Configuring and organizing a mental model	1.45	.44	3.01	.64	1.56	.67	50	.000
3. Working mathematically	1.23	.32	2.45	.78	1.23	.76	50	.000
4. Explaining the answer	1.02	.10	2.00	.92	.98	.92	50	.000
Overall mean score (Total/4)	1.20	0.28	2.56	0.75	1.36	0.74		

The result from test item 1(see annexure 1) revealed that there was difference in students' ability to comprehend the competency based problem between the pre-intervention mean=1.11(SD=0.27) compared to the post intervention mean=2.76(SD=0.67) which showed an increase of mean=1.56 (SD=0.67). The "configuring and organizing a mental model" which required students to translate real world problem to mathematical problem using diagrams and table also observed increased mean of 1.56 (SD=0.67) from baseline score (mean=1.45, SD= 0.44) to post intervention score (mean= 3.01, SD=0.64). Upsurge of mean = 1.23 (SD=0.76) from pre-intervention (mean=1.23, SD= 0.32) to post intervention (mean=2.45, SD=0.78) was observed in the ability if students in working mathematically phase. Moreover, the study indicated that there was mean difference of 0.98 (SD=0.92) between the pre-intervention (Mean=1.02, SD=0.10) and post intervention (mean=2.0, SD=0.92) score of "explaining the answer". The p-value for all the paired sample test was less than 0.001 indicating that the findings were statistically significant. Moreover, the result further revealed that the overall mean score of post intervention (mean=2.56, SD=0.75) is higher than that of the pre-intervention (mean=1.20, SD=0.28) which indicated a significant improvement in the students' ability to solve competency based items attributing the use of mathematical model.

Table 3: Paired Samples T-test (Question 2)

	Pre-intervention		**Post intervention**		**Paired Differences**			
	Mean	Std. Deviation	Mean	Std. Deviation	Mean	Std. Deviation	df	Sig. (2-tailed)
1.Comprehending the problem	1.11	0.27	2.87	0.68	1.57	0.62	50	0.000
2. Configuration and organizing a mental model	1.57	0.49	3.14	0.69	1.57	0.68	50	0.000
3. Working mathematically	1.38	0.49	2.59	0.77	1.21	0.74	50	0.000
4. Explaining the answer	1.19	0.35	2.20	0.86	1.01	0.85	50	0.000
Mean of mean(total/4)	1.31	0.4	2.7	0.75	1.34	0.72		

Similar to the result of test item 1, the result from test item 2(see annexure 1) indicated that there was a significant improvement in the students' ability to solve competency based item which was attributed to the use of mathematical model. There was a significant difference in the overall scores of pre-intervention (mean=1.31, SD=0.4) and post intervention (mean=2.7, SD=0.75). The study have revealed that use of mathematical model has improved "comprehending problem" score of pre-intervention mean of 1.11(SD=0.27) to the post intervention mean of 2.87(SD=0.68); "configuring and organizing a mental model" from baseline score of mean=1.45 (SD= 0.44) to post intervention score mean= 3.01 (SD=0.64); "working mathematically" from pre-intervention mean=1.38 (SD= 0.49) to post intervention mean=2.59 (SD=0.77) and; "explaining the answer" from pre-intervention mean=1.19 (SD=0.35) to post intervention mean=2.20 (SD=0.86). The significance of two-tailed test (p-value) for all the components was less than 0.001 indicating that the findings were statistically significant.

5.2 Discussion

The purpose of this action research was to explore whether teaching of mathematical model can help learners solve competency based problems. This study also looked for possible causes of difficulty of students in solving competency based word problems.

The first finding revealed that students have problem in comprehending the question or some read the questions partially. It was relevant to that of Pearce et al. (2013) which revealed that students struggled with reading and understanding the problems which is attributed in lack of success in solving word problems. The students' ability to read and comprehend the mathematical text remains at the core of solving CBAT problems. Searching for key words was found to have little problem solving success but this kind of strategies are taught and re-taught to struggling students in spite of their lack of success (Pearce et al., 2013). Letting students to learn how to paraphrase and letting them write or rewrite questions in their own language was observed to be helpful. The pupils need to understand the given question by making reflection on certain aspects related with their own thinking.

The second finding states that the inability to translate real world problem to mathematical problem one of the causes of difficulties of students in solving CBAT items. The PISA-D report of Bhutan uncovered similar causes of difficulty which states that items related to the mathematical process "formulating situations mathematically" were the most difficult ones (BCSEA, 2019). The wrong interpretation of real world problem leads to wrong choice of strategies to work mathematically. Verschaffel and de Corte (2000) states that school mathematics was taught in such a way that school children, after receiving instruction in mathematics, regarded mathematics purely as an academic discipline divorced from real-world problem solving (in Wu & Adams, 2006). Similarly, the lack of success in solving competency based items is because our students are taught mathematics as an academic discipline following the prescribed textbook and curriculum. Moreover, students failed to draw connection between the answers they derived from mathematizing and the answer expectation from the real world problem. Therefore, explaining the answer can help students answer the CBAT wholly and with reasoning.

Moreover, the study found that mathematical modelling is statistically significant and helped students master every component of the model employed for this study. The mean differences in the baseline and post intervention data showed radical improvement in ability to solve competency based items by students. This result indicated that guiding students to solve problems step by step can increase their deep processing of information (Hsia, Lin, Chen & Peng, 2018). The findings of Lowe, Carter and Cooper (2018) was supported by this study. It was learnt that the mathematical modelling can make a clear connection to reality and mathematics as an academic discipline. It provides an engaging approach to learning mathematics and shows mathematics to be a body of interconnected knowledge rather than a series of discrete topics. Such kind of benefits of mathematic modelling makes learning authentic and realistic.

6. Conclusion

A central problem in mathematics education is that the educators want students to become proficient problem solvers but even after 20 years of researches and reforms, many students still do rote learning and algorithmic thinking (Lithner, 2006). Similarly, the mathematics workbooks and textbooks contain more of contextualized/conventional problems but the Mathematics Competency Based Assessments for class VI board examination conducted by Bhutan Council of School Examination and Assessment (BCSEA) consists more of competency based items. The discrepancy between the types of problems we teach in the class (mostly conventional problems) and our expectation from our students to solve competency based problems can be a factor behind learning difficulties in mathematics. Hence, the findings from this study should provide an alternative of using mathematical model to make students proficient problem solver.

Mathematical modelling can be a process of unlearning and re-learning as they learn to appreciate how they can learn and apply mathematics in a more holistic way (Eric, Dawn, Wanty & Seto, 2012). The four phases (*comprehending the problem - configuring and organizing a mental model - working mathematically – explaining the answer*) progresses in sequence and succeed one another linearly in solving a problem. The success of former phases determines the scope of successful of outcome of following phases. The knowledge on this model can help students' mathematical leaning more meaningful (Blum & Ferri, 2009) and teachers obtain the patterns of students' problem-solving competence (Singer & Voica, 2013).

Mathematical modelling is also difficult for teachers since real world knowledge is needed and teaching becomes more open and less predictable (Blum & Ferri, 2009). Therefore, it has appeared that a thorough study needs to be carried out with Bhutanese teachers to see the competence and knowledge of mathematical knowledge which this paper couldn't focus.

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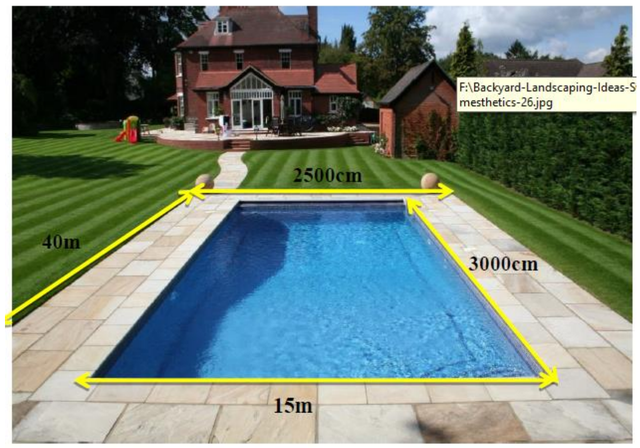
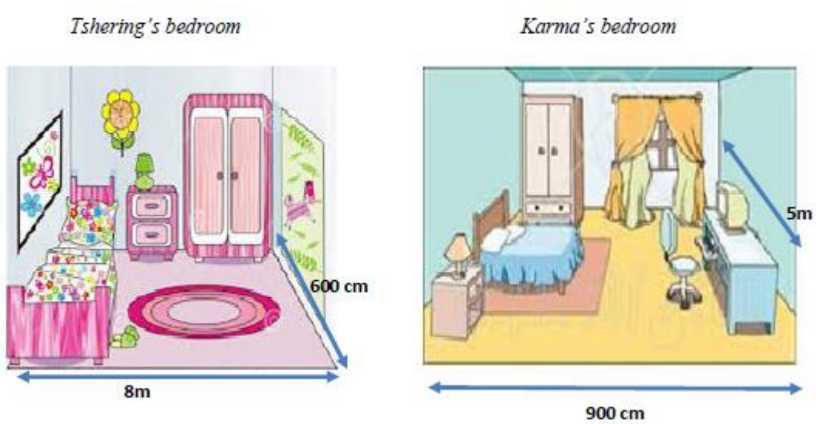
Annexure 1: Competency based item used for baseline and post intervention data collection

Question 1: The white area around the swimming pool in the diagram below is called the deck or walkway. Find the area of the deck around the swimming pool.



Question 2: Figure below shows the dimensions of Tshering's and Karma's bedroom.



Whose bedroom has larger area? Find out by how much more? Show your work.

Annexure 2: Rubrics

	Comprehending the problem	Configuration and organizing a mental model	Working mathematically	Explaining the answer
Exemplary 4	The students can read the question precisely and demonstrate the understanding of the situation clearly.	The student can recognize all the given information and make assumptions. The diagrams/words/tables were used appropriately.	The student uses appropriate procedures and shows step-by-step how he/she arrived at the answer(s).	The student can link the mathematical answer to the real world question and answers correctly.
Proficient 3	The student understood the problem and his/her strategy works.	The student can recognize all the given information. The diagrams/words/tables were used.	The student used correct procedures and shows steps how he/she arrived at the answer(s). It is evident as student used math language and/or notation throughout his/her work.	The student can answer mathematically but failed to link the answer to the real world question.
Emerging 2	The student only understood part of the problem. He/she needs help in understanding how to finish the problem.	The student can recognize the given information party. The diagrams/words/tables were not clear.	The student uses procedures but steps are not clear how he/she arrived at the answer(s). The student used some math language and/or math notation.	The student can answer partly and failed to link the answer to the real world question.
Not Evident 1	The student did not understand the problem. The student needs help in understanding the problem and choosing a strategy to solve the problem.	The student cannot recognize any given information. The diagrams/words/tables were not used.	The student couldn't use appropriate procedures and show step-by-step how he/she arrived at the answer(s). The student used no math language and/or math notation.	The student cannot answer correctly.

Annexure 3: Mathematical Modelling worksheet