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| RESEARCH ARTICLE

On Absolute Valued Algebras with a Central Algebraic Element and Satisfying Some Identities

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| ABSTRACT

In (Moutassim, n.d), we have proven that if A is an absolute valued algebra containing a nonzero central algebraic element, then A is a pre-Hilbert algebra. Here we show that A is finite dimensional in the following cases:
1) A satisfies $(x^2,x,x) = 0$ or $(x,x,x^2) = 0$
2) A satisfies $(x^2,x^2,x) = 0$ or $(x,x^2,x^2) = 0$.
In these cases A is isomorphic to $\mathbb{R},\mathbb{C},\mathrm{H}$ or O.

| KEYWORDS

Absolute valued algebra, pre-Hilbert algebra, algebraic element, central element

| ARTICLE INFORMATION

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1. Introduction

Absolute valued algebras are those real or complex algebras A satisfying $||xy|| = ||x||.||y||$ for a given norm $||.||$ on A, and $x,y \in A$. It it well known that any familiar identity in an absolute valued algebras as commutativity [Urbanik, 1960] or power associativity [Wright, 1953; El-Mallah, 1980] carry away finite dimensionality. Albert's paper [1947] contains a fundamental result asserting that any finite dimensional absolute valued algebras has dimension n = 1, 2,4 or 8 and is isotopic to one of classical (unital) absolute valued algebras $\mathbb{R},\mathbb{C},\mathrm{H}$ or O. El-Mallah and Micali showed that any flexible absolute valued algebras is finite dimensional [El-Mallah, 1981]. Next, El-Mallah showed that for a finite dimensional absolute valued algebra A, flexibility and identity $(x,x,x) = 0$ (where $(.,.,.)$ means associator) coincide [El-Mallah, 1987]. Recently the study of absolute valued algebras with weakly identities as $(x^2,x,x) = 0$, $(x,x,x^2) = 0, (x^2,x^2,x) = 0$ or $(x,x^2,x^2) = 0$, becomes of actuality. It is shown that any absolute valued algebras with a central idempotent and satisfying $(x^2,x,x) = 0$ or $(x,x,x^2) = 0$ is finite dimensional and isomorphic to $\mathbb{R},\mathbb{C},\mathrm{H}$ or O [El-Mallah, 2001]. Urbanik and Wright proved in 1960 that all unital absolute valued algebras are classified by $\mathbb{R},\mathbb{C},\mathrm{H}$ and O [10]. It is easily seen that the one-dimensional absolute valued algebras are classified by $\mathbb{R}$, and it is well-known that the two-dimensional absolute valued algebras are classified by $\mathbb{C}$, $\mathbb{C}^*$, $^*\mathbb{C}$ or $\overset{*}{\mathbb{C}}$ (the real algebras obtained by endowing the space $\mathbb{C}$ with the product $x * y = \bar{x}y$, $x * y = x\bar{y}$, and $x * y = \bar{x}\,\bar{y}$ respectively) [Rodriguez, 1994]. It is natural to study those absolute valued algebras by replacing the original assumption **central idempotent** by a weaker one **central algebraic element**, we prove that, if A is an absolute valued real algebra containing a central algebraic element a and satisfying one of the following identities $(x^2,x,x) = 0$, $(x,x,x^2) = 0, (x^2,x^2,x) = 0$ or $(x,x^2,x^2) = 0$. Then A is finite dimensional and isomorphic to $\mathbb{R},\mathbb{C},\mathrm{H}$ or O (theorems 3.1, 3.2, 3.3, 3.4, 3.5 and 3.6) this result is an important generalization of a results given in [Chandid, 2008] and [El-Mallah, 2001].

In section 2 we introduce the basic tools for the study of absolute valued algebras containing a central algebraic element. We also give some properties related to central algebraic element satisfying some restrictions on commutativity (proposition 2.6 and lemma 2.7). Moreover, the section 3 is devoted to classify all absolute valued algebras with a central algebraic element and satisfying one of the following identities $(x^2,x,x) = 0$, $(x,x,x^2) = 0, (x^2,x^2,x) = 0$ or $(x,x^2,x^2) = 0$.

The paper ends with the following main results:
Theorem. Let A be an absolute valued algebra containing a nonzero central algebraic element. Then the following assertions are equivalent:
1) A satisfies $(x^2, x, x) = 0$ or $(x, x, x^2) = 0$,
2) A satisfies $(x^2, x^2, x) = 0$ or $(x, x^2, x^2) = 0$,
3) A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

2. Notation and Preliminaries Results
In this paper all the algebras are considered over the real numbers field $\mathbb{R}$.
Definition 2.1 Let B be an arbitrary algebra.
i) B is called a division algebra if the operators $\mathrm{L_x}$ and $\mathrm{R_x}$ of left and right multiplication by x are bijective for all $\mathrm{x} \in \mathrm{B} \setminus \{0\}$.
ii) We say that B is algebraic, if for every x in B, the subalgebra $\mathrm{B}(x)$ of B generated by x is finite dimensional.
iii) We mean by a nonzero central element in B, a nonzero element which commute with all elements of the algebra B.
iv) B is called a normed algebra (respectively, absolute valued algebra) if it is endowed with a space norm: $\| . \|$ such that $\|xy\| \leq \|x\|\|y\|$ (respectively, $\|xy\| = \|x\|\|y\|$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{B}$).
v) B is called a pre-Hilbert algebra if it is endowed with a space norm comes from an inner product (./.) such that

$$(./.) : \mathrm{B} \times \mathrm{B} \longrightarrow \mathbb{R}$$
$$(\mathrm{x}, \mathrm{y}) \mapsto \frac{1}{4}(\|\mathrm{x}+\mathrm{y}\|^2 - \|\mathrm{x}-\mathrm{y}\|^2)$$

The most natural examples of absolute valued algebras are $\mathbb{R}, \mathbb{C}, \mathrm{H}$ (the algebra of Hamilton quaternion) and O (the algebra of Cayley numbers) with norms equal to their usual absolute values [El-Mallah, 2001] and [Urbanik, 1960].

We need the following relevant results:

Theorem 2.2 [Moutassim, n.d] The norm of any absolute valued algebra containing a nonzero central algebraic element comes from an inner product.

Theorem 2.3 [Chandid, 2001] Any absolute valued algebra A with a central idempotent satisfying $(x^2, x, x) = 0$, $(x, x, x^2) = 0, (x^2, x^2, x) = 0$ or $(x, x^2, x^2) = 0$ for all $x \in A$ is finite dimensional and is isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

Theorem 2.4 [Urbanik, 1960] A commutative absolute valued algebra is isomorphic to $\mathbb{R}$, $\mathbb{C}$ or $\overset{*}{\mathbb{C}}$

Theorem 2.5 [Rodriguez, 1994] The norm of any absolute valued algebra A with left unit A comes from an inner product and satisfying $(ab/c) = -(b/ac)$ and $a(ab) = -||a||^2 b$ for all $a, b, c \in A$ with a orthogonal to e.

we give some conditions imply that A is an inner product space.
Proposition 2.6 [8] Let A be an absolute valued algebra containing a central element a and let x be a element in A. If x is orthogonal to a in the inner product space $[a, x]$, then the following are equivalent:
1) $x^2 a^2 = a^2 x^2$,
2) $x^2 = -||x||^2 a^2$,
3) A is an inner product space.

Lemma 2.7 Let A be an absolute valued algebra containing a nonzero central algebraic element a. Then
$xy + yx = 2(x|y)a^2$ for all $x, y \in \{a\}^{\perp}$.
Proof. By theorem 2.2, A is an inner product space. We have $x, y \in \{a\}^{\perp}$, then $(x+y)^2 = -||x+y||^2 a^2$ (proposition 2.6), hence $xy + yx = -2(x|y)a^2$.

3. Main Results
3.1. Absolute Valued Algebras Satisfying $(x^2, x, x) = 0$ or (x, x, x^2)
In this section we prove that if A is an absolute valued algebra with a central element a and satisfying $(x^2, x, x) = 0$ or $(x, x, x^2) = 0$. Then A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

Theorem 3.1 Let A be an absolute valued algebra containing a central algebraic element a and satisfying $(x^2, x, x) = 0$ for all $x \in A$. If a and a^2 are linearly independent, then $A(a, a^2)$ is isomorphic to $\mathbb{C}$.
Proof. By theorem 2.2, A is an inner product space. Let $d = a^2 - (a/a^2)a$, we have $(d/a) = 0$, by lemma 2.7

$$d^2 = -||d||^2 a^2 = -(1 - (a/a^2)^2)a^2$$

That is

$$-(1 - (a/a^2)^2)a^2 = (a^2 - (a/a^2)a)^2$$
$$= (a^2)^2 - 2(a/a^2)aa^2 + (a/a^2)^2 a^2$$

$$= (a^2a)a - 2(a/a^2)aa^2 + (a/a^2)^2a^2$$
$$= (a^2a - 2(a/a^2)a^2 + (a/a^2)^2a)a$$

This gives $\quad -(1 - (a/a^2)^2)a = a^2a - 2(a/a^2)a^2 + (a/a^2)^2a$

So $\quad a^2a = 2(a/a^2)a^2 - a$

Hence $A(a,a^2)$ is a two-dimensional commutative sub-algebra of A, thus $A(a,a^2)$ is isomorphic to $\mathbb{C}$ or $\overset{*}{\mathbb{C}}$ (theorem 2.4). If $A(a,a^2)$ is isomorphic to $\overset{*}{\mathbb{C}}$, then there exist a basis $\{f,j\}$ of $A(a,a^2)$ such that $f^2 = f,\ j^2 = -f\ \ and\ jf = fj = -j$. Since

$$(j^2,j,j) = -(f,j,j) = -(fj)j + fj^2 = -f - f = -2f \neq 0$$

Which absurd, therefore $A(a,a^2)$ is isomorphic to $\mathbb{C}$.

From the last result we conclude there exists a nonzero idempotent $e \in A$ and a nonzero element $i \in A$ such that $e^2 = e,\ ie = ei\ and\ i^2 = -e$. We put $a = \alpha e + \beta i$ with $\alpha,\beta \in \mathbb{R}$ $(\alpha^2 + \beta^2 = 1)$. Then we get the following result:
Theorem 3.2 Let A be an absolute valued algebra containing a central algebraic element a and satisfying $(x^2,x,x) = 0$ for all $x \in A$, then A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

Proof. By theorem 2.2, A is an inner product space. Let $x \in \{a,a^2\}^{\perp}$ be a norm one element, we have the following two cases:
1) If a and a^2 are linearly independent, then $A(a,a^2)$ is isomorphic to $\mathbb{C}$ (theorem 2.4). We put $a = \alpha e + \beta i$ (notation above) and led $d = ia = \alpha i - \beta e$, we have $(d/a) = (ia/a) = (i/e) = 0$, then $dx = -xd$ (Lemma 2.7). Since $ax = xa$, we obtain

$$\alpha ex + \beta ix = \alpha xe + \beta xi \quad and \quad -\beta ex + \alpha ix = -\beta xe + \alpha xi$$

From these equalities, we get $\quad \beta\alpha ex + \beta^2 ix = \beta\alpha xe + \beta^2 xi \quad (1)$

and $\quad -\alpha\beta ex + \alpha^2 ix = -\alpha\beta xe + \alpha^2 xi \quad (2)$

Adding the two equalities (1) and (2), we obtain $ix = xi$ $(\alpha^2 + \beta^2 = 1)$. According to proposition 2.6 $x^2 = -i^2 = e$. Since $x \in \{a,a^2\}^{\perp}$ and $ax = xa$, then $e = x^2 = -a^2$. That is, $e = -(\alpha^2 - \beta^2)e - 2\alpha\beta i$, this implies $\alpha = 0$ or $\beta = 0$.
a and a^2 are linearly independent, thus $\beta \neq 0$, therefore $\alpha = 0$. Which means that $(a/a^2) = (a/e) = 0$.

On the other hand,
$$\begin{aligned} 0 &= ((e+x)^2, e+x, e+x) \\ &= (2e, e+x, e+x) \\ &= (e,e,x) + (e,x,e) + (e,x,x) \\ &= (e,e,x) + (e,x,e) \qquad ((e,x,x) = (x^2,x,x) = 0) \\ &= ex - e(ex) + (ex)e - e(xe) \\ &= -xe - e(ex) + (ex)e + e(ex) \qquad (\text{by lemma 2.7, } ex + xe = 0) \end{aligned}$$

This implies $(ex)e = xe$, thus $ex = x$. Since $ev = ve = v$ for all $v \in A(a,a^2)$, then $ey = y$ for all $y \in A$. Hence e is a left unit of A. Moreover
$$\begin{aligned} 0 &= ((a+x)^2, a+x, a+x) \\ &= (ax, a+x, a+x) \\ &= (ax,a,a) + (ax,a,x) + (ax,x,a) + (ax,x,x) \end{aligned}$$

We replace x by $-x$, we get $\quad (ax,a,a) + (ax,x,x) = 0 \quad (3)$

and $\quad (ax,a,x) + (ax,x,a) = 0 \quad (4)$

So (4) gives $\quad ((ax)a)x - (ax)^2 + ((ax)x)a - (ax)^2 = 0$

That is $\quad (a(ax))x + ((ax)x)a = 2(ax)^2$

As $(a/e) = 0$ and theorem 2.5, we get $-x^2 + ((ax)x)a = 2(ax)^2 \quad (5)$

We have $\quad (ax/a) = (x/e) = 0$, then $(ax)^2 = -a^2 = e$,

So (5) gives $\quad ((ax)x)a = -3e$

Which absurd, $||((ax)x)a|| = 1$ and $||-3e|| = 3$. Therefore $x = 0$, in this case $A = A(a,a^2)$ is isomorphic to $\mathbb{C}$.
2) If a and a^2 are linearly dependent, then a is a nonzero central idempotent and the theorem 2.3 completes the proof.

Similarly, we can get all preceding results if A satisfies (x,x,x^2)
Theorem 3.3 Let A be an absolute valued algebra containing a central algebraic element a and satisfying (x,x,x^2) for all $x \in A$, then A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

3.2 Absolute Valued Algebras Satisfying $(x^2,x^2,x) = 0$ *or* $(x,x^2,x^2) = 0$

In this section we prove that if A is an absolute valued algebra containing a central element a and satisfying $(x^2,x^2,x) = 0$ or $(x,x^2,x^2) = 0$. Then A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

Theorem 3.4 Let A be an absolute valued algebra containing a central algebraic element a and satisfying $(x^2,x^2,x) = 0$ for all $x \in A$. If a and a^2 are linearly independent, then $A(a,a^2)$ is isomorphic to $\mathbb{C}$.
Proof. By theorem 2.2, A is an inner product space. Let $d = a^2 - (a/a^2)a$, $(d \neq 0)$, we have $(d/a) = 0$, by proposition 2.6

$$d^2 = -||d||^2a^2 = -(1 - (a/a^2)^2)a^2$$

That is $\quad -(1 - (a/a^2)^2)a^2 = (a^2 - (a/a^2)a)^2$

$$-a^2 + (a/a^2)^2a^2 = (a^2)^2 - 2(a/a^2)aa^2 + (a/a^2)^2a^2$$

This gives $$(a^2)^2 = 2(a/a^2)aa^2 - a^2 \quad (6)$$

- If $(a/a^2) = 0$, then $(a^2)^2 = -a^2$ and $(a^2)^2a = -a^2a$

$$a^2(a^2a) = -a^2 = (a^2)^2$$

Hence $aa^2 = a$, which means that $A(a, a^2)$ is a two dimensional commutative sub-algebra of A.
Let $c = aa^2 - (a/aa^2)a$, we have $(c/a) = 0$, by proposition 2.6,

$$c^2 = -||c||^2a^2 = -(1 - (a/aa^2)^2)a^2$$

- If $||c|| = 0$, then $aa^2 = \pm a$. That is $(a^2)^2 = 2(a/a^2)aa^2 - a^2 = \pm 2(a/a^2)a - a^2$

This implies that $A(a, a^2)$ is a two dimensional commutative sub-algebra of A.
Assuming that $(a/a^2) \neq 0$ and $||c|| \neq 0$. Since $(d^2, d^2, d) = 0$, then $(a^2, a^2, a^2) = 0$ thus $(a^2)^2a^2 = a^2(a^2)^2$
So (6) gives $(aa^2)a^2 = a^2(aa^2)$, moreover

$$\begin{aligned} dc &= (a^2 - (a/a^2)a)(aa^2 - (a/aa^2)a) \\ &= a^2(aa^2) - (a/a^2)a(aa^2) - (a/aa^2)aa^2 + (a/a^2)(a/aa^2)a^2 \\ &= (aa^2)a^2 - (a/a^2)a(aa^2) - (a/aa^2)aa^2 + (a/a^2)(a/aa^2)a^2 \\ &= cd \end{aligned}$$

And since $||c||^2d^2 = ||d||^2c^2$, then $||c||d = ||d||c$ or $||c||d = -||d||c$. We conclude that

$$||d||aa^2 = ||c||a^2 + ((a/aa^2)||d|| - (a/a^2)||c||)a$$

Or

$$||d||aa^2 = ||c||a^2 + ((a/aa^2)||d|| - (a/a^2)||c||)a$$

Therefore $A(a, a^2)$ is a two-dimensional commutative sub-algebra of A, thus $A(a, a^2)$ is isomorphic to $\mathbb{C}$ or $\overset{*}{\mathbb{C}}$ (theorem 2.4). If $A(a, a^2)$ is isomorphic to $\overset{*}{\mathbb{C}}$,that is, there exist a basis $\{f, j\}$ of $A(a, a^2)$ such that $f^2 = f,\ j^2 = -f\ \ and\ jf = fj = -j$. Since

$$(j^2, j^2, j) = (f, f, j) = fj - f(fj) = -j - j = -2j \neq 0$$

Which absurd, therefore $A(a, a^2)$ is isomorphic to $\mathbb{C}$.

From the last result we conclude there exists a nonzero idempotent $e \in A$ and a nonzero element $i \in A$ such that $e^2 = e,\ ie = ei\ and\ i^2 = -e$. We put $a = \alpha e + \beta i$ with $\alpha, \beta \in \mathbb{R}\ (\alpha^2 + \beta^2 = 1)$. Then we get the following result:
Theorem 3.5 Let A be an absolute valued algebra containing a central algebraic element a and satisfying $(x^2, x^2, x) = 0$ for all $x \in A$, then A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

Proof. By theorem 2.2, A is an inner product space. Let $x \in \{a, a^2\}^{\perp}$ be a norm one element, we have the following two cases:
1) If a and a^2 are linearly independent, then $A(a, a^2)$ is isomorphic to $\mathbb{C}$ (theorem 2.4). We put $a = \alpha e + \beta i$ (notation above) and $d = ia = \alpha i - \beta e$, we have $(d/a) = (ia/a) = (i/e) = 0$, then $dx = -xd$ (Lemma 2.7). Since $ax = xa$, we obtain

$$\alpha ex + \beta ix = \alpha xe + \beta xi \ \ and\ -\beta ex + \alpha ix = -\beta xe + \alpha xi$$

From these equalities, we get $$\beta\alpha ex + \beta^2 ix = \beta\alpha xe + \beta^2 xi \quad (7)$$
and $$-\alpha\beta ex + \alpha^2 ix = -\alpha\beta xe + \alpha^2 xi \quad (8)$$
Adding the two equalities (7) and (8), we obtain $ix = xi$ $(\alpha^2 + \beta^2 = 1)$. According to proposition 2.6 $x^2 = -i^2 = e$. Since $x \in \{a, a^2\}^{\perp}$ and $ax = xa$, then $e = x^2 = -a^2$. That is, $e = -(\alpha^2 - \beta^2)e - 2\alpha\beta i$, this implies $\alpha = 0$ or $\beta = 0$.
But we have, a and a^2 are linearly independent, thus $\beta \neq 0$, therefore $\alpha = 0$. Which means that $(a/a^2) = (a/e) = 0$.
On the other hand, using lemma 2.7, $ex + xe = 0$. So

$$\begin{aligned} 0 &= ((e + x)^2, (e + x)^2, e + x) \\ &= (2e, 2e, e + x) \\ &= (e, e, x) \\ &= ex - e(ex) \end{aligned}$$

this implies $ex = x$. Since $ev = ve = v$ for all $v \in A(a, a^2)$, then $ey = y$ for all $y \in A$. Hence e is a left unit of A.
Moreover

$$\begin{aligned} 0 &= ((a + x)^2, (a + x)^2, a + x) \\ &= (ax, ax, a + x) \qquad (x^2 = -a^2) \\ &= (ax, ax, a) + (ax, ax, x) \end{aligned}$$

We replace x by $-x$, we get $$(ax, a, a) = 0 \quad (9)$$
and $$(ax, a, x) = 0 \quad (10)$$
So (10) gives $(ax)^2 - ((ax)a)x = 0$
That is $(a(ax))x = (ax)^2$
As $(ax/a) = (x/e) = 0$, thus $(ax)^2 = -a^2 = e$. And by theorem 2.5, we get $-x^2 = (ax)^2 = e = x^2$
Therefore $x = 0$, in this case $A = A(a, a^2)$ is isomorphic to $\mathbb{C}$.
2) If a and a^2 are linearly dependent, then a is a nonzero central idempotent and the theorem 2.3 completes the proof.

Similarly, we can get all preceding results if A satisfies (x, x^2, x^2)
Theorem 3.6 Let A be an absolute valued algebra containing a central algebraic element a and satisfying (x, x^2, x^2) for all $x \in A$, then A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

4. Conclusion

We have the following classical results:
Theorem. Let A be an absolute valued algebra containing a nonzero central algebraic element. Then the following assertions are equivalent:
1) A satisfies $(x^2, x, x) = 0$,
2) A satisfies $(x, x, x^2) = 0$,
3) A satisfies $(x^2, x^2, x) = 0$,
4) A satisfies $(x, x^2, x^2) = 0$,
5) A is finite dimensional and isomorphic to $\mathbb{R}, \mathbb{C}, \mathrm{H}$ or O.

Based on the findings of this article, the following conclusions can be drawn:
1) In general, if A is a real absolute valued algebra containing a nonzero central algebraic element, then, A is a pre-Hilbert algebra. It may be conjectured that every absolute valued algebra containing a nonzero central element is pre-Hilbert algebra.
2) Note that, central idempotent is a central element. The reciprocal case does not hold in general, and the counter example is given [Benslimane, 2011].
3) We classify all real absolute valued algebra containing a nonzero central algebraic element and satisfying $(x^2, x, x) = 0, (x, x, x^2) = 0, (x^2, x^2, x) = 0$ *or* $(x, x^2, x^2) = 0$. In future work, it is intended to study the finite dimensional real algebras containing a nonzero central element.

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