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Extra Skolem Difference Mean Labeling of Various Graphs

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ABSTRACT

Let graph G=(V(G),E(G)) attains a Skolem difference mean labeling with p vertices and q edges is said to be an extra Skolem difference mean labeling of graph G if all the labels of the vertices are odd. The graph which attains an extra Skolem difference mean labeling is called an extra Skolem difference mean graph. We obtain an extra Skolem difference mean labeling for Comb graph, Twig of a path P_n, H graph of a path P_n, K_1,2*K_(1,n) graph, K_1,3*K_(1,n) graph, m- Join of H_n, P_n⊙K_(1,m) graph , HSS(P_n) graph, H⊙〖mK〗_1-graph of a path P_n.

1. Introduction

We consider finite, connected and undirected graph. We consider graph G having set of vertices $V(G)$ and set of edges $E(G)$. An excellence reference on this subject is the survey by J. A. Gallian [4]. Skolem difference mean labeling was introduced by K. Murugan and A. Subramanian in [6]. Selvi, Ramya and Jeyanthi [7] define an extra Skolem difference mean labeling of graphs. We refer Gross and Yellen [5], for all kinds of definitions and notations.

Definition: Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. An injective mapping $f: V \rightarrow \{1,2,3,\dots,p+q\}$ is called a **Skolem difference mean labeling** if which includes an bijective edge labeling $f^*: E \rightarrow \{1,2,3,\dots,q\}$ defined by $f^*(e = uv) = \frac{|f(u)-f(v)|}{2}$ if $|f(u) - f(v)|$ is even otherwise $\frac{|f(u)-f(v)|+1}{2}$, if $|f(u) - f(v)|$ is odd and the graph is called a Skolem difference mean graph .[1]

Definition: - Let graph $G = (V(G), E(G))$ attains a Skolem difference mean labeling with p vertices and q edges is said to be an **extra Skolem difference mean labeling** of graph G if all the labels of the vertices are odd. The graph which attains an extra Skolem difference mean labeling is called an extra Skolem difference mean graph.[2]

SOME EXISTING RESULTS:

- The graph $T < K_{1,n_1}: K_{1,n_2}: \dots K_{1,n_m} >$ is an extra Skolem difference mean graph.[7]
- Path is an extra Skolem difference mean graph. [7]
- The graph $T < K_{1,n}: K_{1,n}: \dots K_{1,n} >$ is an extra Skolem difference mean graph[7].
- The graph $T < K_{1,n_1} \circ K_{1,n_2} \circ \dots K_{1,n_{m-1}} \circ K_{1,n_m} >$ is an extra Skolem difference mean graph.[7]
- T_p −tree is an extra Skolem difference mean graph. [2]
- $< T\hat{o}K_{1,n} >$ is an extra Skolem difference mean graph. [2]
- The Caterpillar graph is an extra Skolem difference mean graph. [2]
- The graph $S_{m,n}$ is an extra Skolem difference mean graph.[2]

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- Star is an extra Skolem difference mean graph. [3]
- $B(m,n)$ is an extra Skolem difference mean graph. [3]
- $B(m,n,k)$ is an extra Skolem difference mean graph. [3]
- Coconut tree $CT(m,n)$ is an extra Skolem difference mean graph.[3]
- F −tree FP_n is an extra Skolem difference mean graph.[3]
- Y −tree is an extra Skolem difference mean graph.[3]

Definition: The **Comb** $P_n \odot K_1$ is the graph obtained from a path P_n by attaching a pendant edge to each vertex of the path..[1]

Definition: The graph obtained from a path P_n by attaching exactly two pendant edges to each internal vertex of the path is called the **Twig** graph.[11]

Definition: The $\boldsymbol{H}$ **graph** of path P_n is obtained from two copies of P_n with vertices $u_1, u_2, \dots, u_n$ & $v_1, v_2, \dots, v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ & $v_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $u_{\frac{n}{2}+1}$ & $v_{\frac{n}{2}}$ if n is even.[10]

Definition: $\boldsymbol{K_{1,2} * K_{1,n}}$ is the graph obtained from $K_{1,2}$ by attaching root of a star $K_{1,n}$ at each pendant vertex of $K_{1,2}$.

Definition: $\boldsymbol{K_{1,3} * K_{1,n}}$ is the graph obtained from $K_{1,3}$ by attaching root of a star $K_{1,n}$ at each pendant vertex of $K_{1,3}$.[1]

Definition: The **corona** $\boldsymbol{G_1 \odot G_2}$ of two graphs G_1 & G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.[10]

Definition: $\boldsymbol{m}$ **−Joins of** $\boldsymbol{H}$ **graph** is a graph where each of H graph denoted by H_{n_1} by an edge e_1 with H graph denoted by H_{n_2}, H graph denoted by H_{n_2} by an edge e_2 with H graph denoted by H_{n_3} and so on with H graph denoted by $H_{n_{m-1}}$ by an edge e_{m-1} with H graph denoted by H_{n_m} such that $n_1 = n_2 = n_m$.[9]

Definition: Let G be a graph. A graph obtained from G by replacing each edge e_i by a H graph in such a way that the ends of e_i are merged with a pendant vertex in P_2 and pendant vertex in P'_2 is called $\boldsymbol{H}$ **super subdivision of** $\boldsymbol{G}$ is denoted by $HSS(G)$, where the H graph is a tree on 6 vertices in which exactly two vertices of degree 3.[8]

2. Main Results

Theorem 2.1.1 The Comb $P_n \odot K_1$ graph is an extra Skolem difference mean graph.

Proof: Let $G = P_n \odot K_1$ with $V(G) = \{u_k, v_k : 1 \le k \le n\}$ and

$E(G) = \{(u_k\, u_{k+1}) : 1 \le k \le n-1\} \cup \{(u_k\, v_k) : 1 \le k \le n\}$.

$\therefore\ |V(G)| = 2n$ & $|E(G)| = 2n - 1$.

Define $f: V(G) \to \{1,2,3,\dots,4n-1\}$,

$$f(u_{2k-1}) = 4n + 3 - 4k \qquad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(u_{2k}) = 4k - 1 \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v_{2k-1}) = 4k - 3 \qquad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(v_{2k}) = 4n + 1 - 4k \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$$

We define edge function $f^*: E(G) \to \{1,2,3 \dots, 2n-1\}$ as follows.

$$f^*(u_k u_{k+1}) = 2n - 2k \qquad 1 \le k \le n-1$$

$$f^*(u_k v_k) = 2n + 1 - 2k \qquad 1 \le k \le n$$

Which is bijective function. Hence Comb $P_n \odot K_1$ graph is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of $P_5 \odot K_1$ is shown in Figure-1.

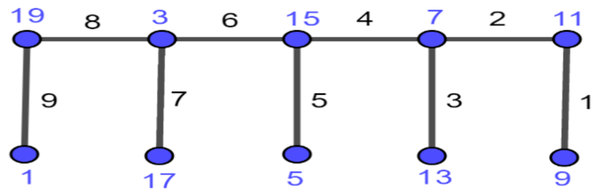

Figure-1 $P_5 \odot K_1$

Theorem 2.1.2 Twig obtained from path $TW(P_n)$ is an extra Skolem difference mean graph $\forall n \geq 3$.

Proof: Let $G = TW(P_n)$

$V(G) = \{u_k : 1 \leq k \leq n\} \cup \{v_k, w_k : 1 \leq k \leq n-2\}$

$E(G) = \{(u_k u_{k+1}) : 1 \leq k \leq n-1\} \cup \{(u_{k+1} v_k) : 1 \leq k \leq n-2\} \cup \{(u_{k+1} w_k) : 1 \leq k \leq n-2\}$

So, $|V(G)| = 3n-4$ & $|E(G)| = 3n-5$.

Define $f : V(G) \to \{1,2,3,\ldots,6n-9\}$,

Case:1 n is even

$f(u_{2k-1}) = 2k-1 \qquad 1 \leq k \leq \frac{n}{2}$

$f(u_{2k}) = 6n-7-2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v_{n-2k-1}) = 5n-7-2k \qquad 1 \leq k \leq \frac{n-2}{2}$

$f(v_{n-2k}) = n-1+2k \qquad 1 \leq k \leq \frac{n-2}{2}$

$f(w_{2k-1}) = 2n-3+2k \qquad 1 \leq k \leq \frac{n-2}{2}$

$f(w_{2k}) = 4n-5-2k \qquad 1 \leq k \leq \frac{n-2}{2}$

Case:2 n is odd

$f(u_{2k-1}) = 2k-1 \qquad 1 \leq k \leq \frac{n+1}{2}$

$f(u_{2k}) = 6n-7-2k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(v_{n-2k}) = 5n-6-2k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(v_{n-2k-1}) = n+2k \qquad 1 \leq k \leq \frac{n-3}{2}$

$f(w_{2k-1}) = 2n-3+2k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(w_{2k}) = 4n-5-2k \qquad 1 \leq k \leq \frac{n-3}{2}$

For both cases we define following edge function, $f^* : E(G) \to \{1,2,3,\ldots,3n-5\}$ as follows.

$f^*(u_k u_{k+1}) = 3n-4-k \qquad 1 \leq k \leq n-1$

$f^*(u_{k+1} v_k) = n-1-k \qquad 1 \leq k \leq n-2$

$f^*(u_{k+1} w_k) = 2n-3-k \qquad 1 \leq k \leq n-2$

Which is bijective function. So, $TW(P_n)$ is an extra Skolem difference Mean graph.

Illustration: An extra Skolem difference Mean labeling of $TW(P_5)$ Figure-2.

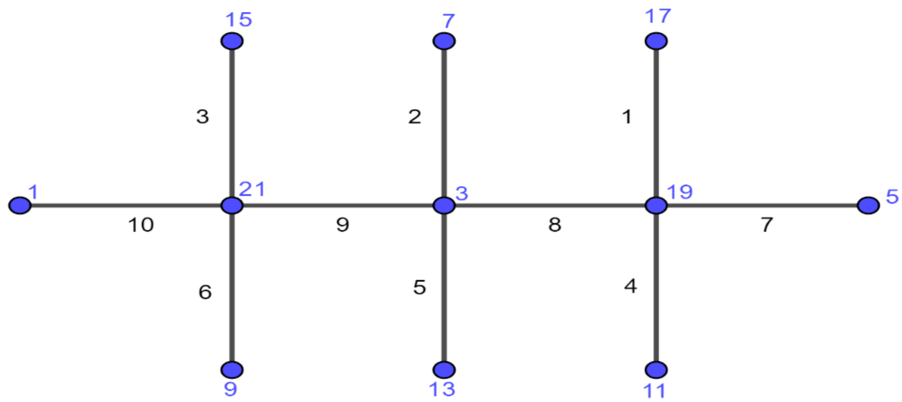

Figure-2 $TW(P_5)$

Theorem 2.1.3 H −graph of a path P_n is an extra Skolem difference mean graph .

Proof: Let G be a H graph of path P_n. P_n be the path $u_1, u_2, \dots, u_n$.we can obtain H −graph by considering two copies of P_n.
Hence we have $V(G) = \{u_k, v_k : 1 \le k \le n\}$ and

$E(G) = \{(u_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{(v_k v_{k+1}): 1 \le k \le n-1\} \cup \{\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right): n \text{ is odd}\}$ OR

$E(G) = \{(u_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{(v_k v_{k+1}): 1 \le k \le n-1\} \cup \{\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right): n \text{ is even}\}$.

$\therefore\ |V(G)| = 2n \,\&\, |E(G)| = 2n - 1$.

Define $f: V(G) \to \{1,2,3, \dots, 4n-1\}$,

Case:1 If n is odd

$f(u_{2k-1}) = 2k - 1 \qquad 1 \le k \le \frac{n+1}{2}$

$f(u_{2k}) = 4n + 1 - 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_{2k-1}) = 3n + 2 - 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v_{2k}) = n + 2k \qquad 1 \le k \le \frac{n-1}{2}$

Case:2 If n is even

$f(u_{2k-1}) = 2k - 1 \qquad 1 \le k \le \frac{n}{2}$

$f(u_{2k}) = 4n + 1 - 2k \qquad 1 \le k \le \frac{n}{2}$

$f(v_{2k-1}) = n - 1 + 2k \qquad 1 \le k \le \frac{n}{2}$

$f(v_{2k}) = 3n + 1 - 2k \qquad 1 \le k \le \frac{n}{2}$

we define following edge function $f^*: E(G) \to \{1,2,3 \dots, 2n-1\}$ as follows.

$f^*(u_k u_{k+1}) = 2n - k \qquad 1 \le k \le n-1$

$f^*(u_k v_k) = n - k \qquad 1 \le k \le n$

$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = n$, n is odd

$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = n$, n is even

Which is bijective function.

Hence H −graph of a path P_n graph is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of H graph of a path P_5 Figure-3.

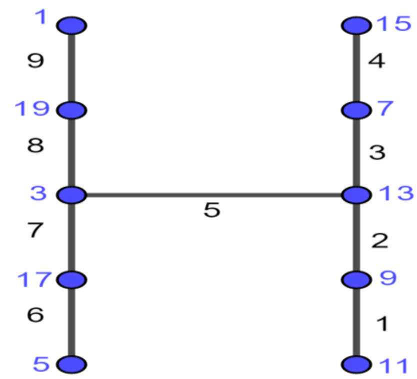

Figure-3 H graph of a path P_5

Theorem 2.1.4 The graph $K_{1,2} * K_{1,n}$ is an extra Skolem difference mean graph for all $n \geq 2$.
Proof: Let $G = K_{1,2} * K_{1,n}$ with $V(G) = \{u, v, w, u_k, v_k : 1 \leq k \leq n\}$

and $E(G) = \{uw, wv, uu_k, vv_k : 1 \leq k \leq n\}$.

Hence $|V(G)| = 2n + 3$ and $|E(G)| = 2n + 2$

Define $f: V(G) \to \{1,2,3, \dots, 4n + 5\}$ by

$f(u) = 1$

$f(v) = 3$

$f(w) = 2n + 5$

$f(u_k) = 2n + 5 + 2k \quad 1 \leq k \leq n$

$f(v_k) = 3 + 2k \quad 1 \leq k \leq n$

We define following edge function $f^*: E(G) \to \{1,2,3, \dots, 2n + 2\}$ as follows.

$f^*(uu_k) = n + 2 + k \quad 1 \leq k \leq n$

$f^*(vv_k) = k \quad 1 \leq k \leq n$

$f^*(wv) = n + 1$

$f^*(uw) = n + 2$

Thus the induced edge labels are distinct from $1,2,3, \dots, 2n + 2$. Hence the graph $K_{1,2} * K_{1,n}$ is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of $K_{1,2} * K_{1,5}$ is shown in Figure-4.

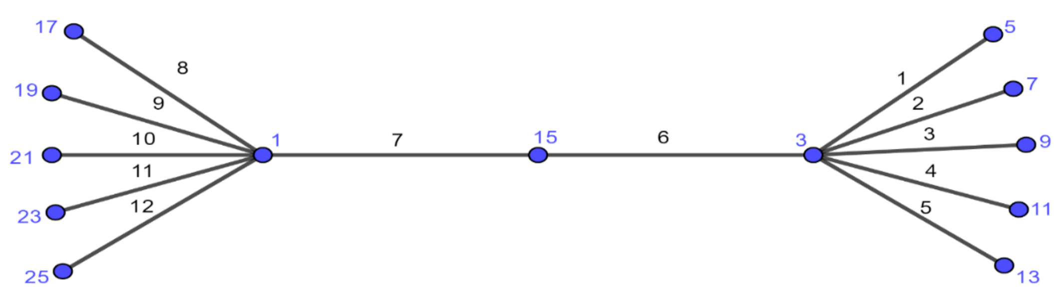

Figure-4 $K_{1,2} * K_{1,5}$

Theorem 2.1.5 The graph $K_{1,3} * K_{1,n}$ is an extra Skolem difference mean graph for all $n \geq 2$.

Proof: Let $G = K_{1,3} * K_{1,n}$ with $V(G) = \{x, u, v, w, u_k, v_k, w_k : 1 \leq k \leq n\}$

and $E(G) = \{ux, xv, xw, uu_k, vv_k, ww_k : 1 \leq k \leq n\}$.

Hence $|V(G)| = 3n + 4$ and $|E(G)| = 3n + 3$

Define $f: V(G) \to \{1,2,3, \dots, 6n + 7\}$ by

$f(u) = 1$

$f(v) = 3$

$f(w) = 2n + 5$

$f(x) = 4n + 7$

$f(u_k) = 6n + 9 - 2k \qquad 1 \leq k \leq n$

$f(v_k) = 4n + 7 - 2k \qquad 1 \leq k \leq n$

$f(w_k) = 2n + 5 - 2k \qquad 1 \leq k \leq n.$

We define following edge function $f^*: E(G) \to \{1,2,3, \dots, 3n + 3\}$ by

$f^*(uu_k) = 3n - k + 4 \qquad 1 \leq k \leq n$

$f^*(vv_k) = 2n - k + 2 \qquad 1 \leq k \leq n$

$f^*(ww_k) = k \qquad 1 \leq k \leq n$

$f^*(ux) = 2n + 3$

$f^*(xv) = 2n + 2$

$f^*(xw) = n + 1$

Thus, the induced edge labels are distinct from $1,2,3, \dots, 3n + 3$. Hence the graph $K_{1,3} * K_{1,n}$ is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of $K_{1,3} * K_{1,6}$ is shown in Figure-5.

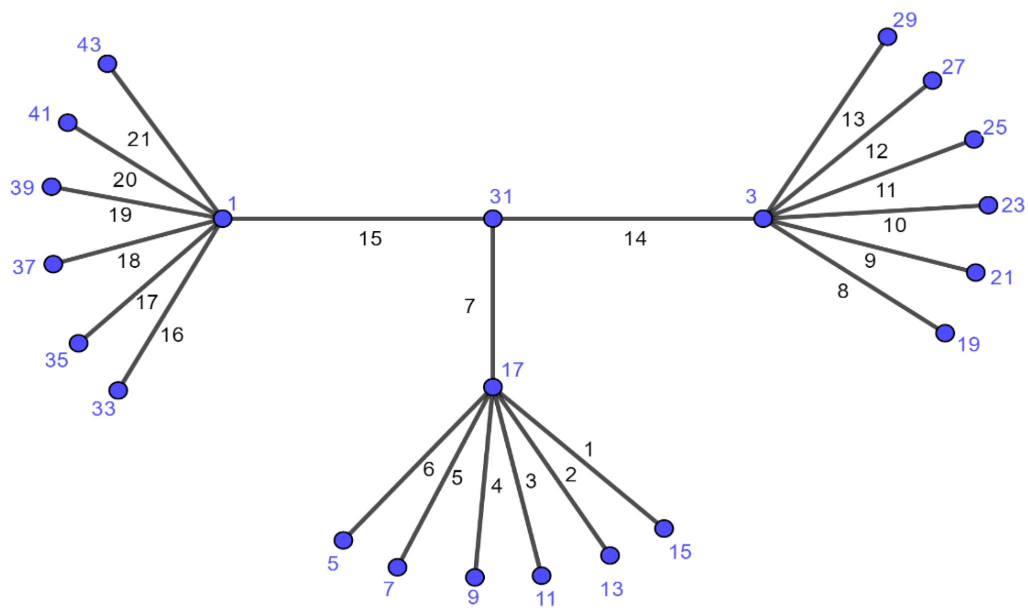

Figure-5 $K_{1,3} * K_{1,6}$

Theorem 2.1.6 m-Join of H_n is an extra Skolem difference mean graph.

Proof: Let $G = m$-Join of H_n

$V(G) = \{u_k, v_k : 1 \leq k \leq n\} \cup \{u'_k, v'_k : 1 \leq k \leq n\} \cup \ldots \cup \{u^m{}_k, v^m{}_k : 1 \leq k \leq n\}$

$E(G) = \{u_k u_{k+1} : 1 \leq k \leq n-1\} \cup \{u'_k u'_{k+1} : 1 \leq k \leq n-1\} \cup \ldots \{u^m{}_k u^m{}_{k+1} : 1 \leq k \leq n-1\} \cup \{v_k v_{k+1} : 1 \leq k \leq n-1\} \cup \{v'_k v'_{k+1} : 1 \leq k \leq n-1\} \cup \ldots \{v^m{}_k v^m{}_{k+1} : 1 \leq k \leq n-1\} \cup \{v_n u'_1\} \cup \{v'_n u''_1\} \ldots \cup \{v^{m-1}{}_n u^m{}_1\}$
$\cup \left\{u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right\} \cup \left\{u'_{\frac{n}{2}+1} v'_{\frac{n}{2}}\right\} \cup \ldots \cup \left\{u^m{}_{\frac{n}{2}+1} v^m{}_{\frac{n}{2}}\right\}$ if n is even.

$E(G) = \{u_k u_{k+1} : 1 \leq k \leq n-1\} \cup \{u'_k u'_{k+1} : 1 \leq k \leq n-1\} \cup \ldots \{u^m{}_k u^m{}_{k+1} : 1 \leq k \leq n-1\} \cup \{v_k v_{k+1} : 1 \leq k \leq n-1\} \cup \{v'_k v'_{k+1} : 1 \leq k \leq n-1\} \cup \ldots \{v^m{}_k v^m{}_{k+1} : 1 \leq k \leq n-1\} \cup \{v_n u'_1\} \cup \{v'_n u''_1\} \ldots \cup \{v^{m-1}{}_n u^m{}_1\}$
$\cup \left\{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right\} \cup \left\{u'_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right\} \cup \ldots \cup \left\{u^m{}_{\frac{n+1}{2}} v^m{}_{\frac{n+1}{2}}\right\}$ if n is odd.

$So, |V(G)| = 2mn + 2n$ & $|E(G)| = 2mn + 2n - 1$.

Define $f : V(G) \cup E(G) \rightarrow \{1,2,3, \ldots, 4mn + 4n - 1\}$ as follows.

Case:1 n is even.

$f(u_{2k-1}) = 2k - 1 \qquad 1 \leq k \leq \frac{n}{2}$

$f(u_{2k}) = 4mn + 4n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v_{2k-1}) = n - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v_{2k}) = 4mn + 3n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(u'_{2k-1}) = 2n - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(u'_{2k}) = 4mn + 2n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v'_{2k-1}) = 3n - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v'_{2k}) = 4mn + n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(u''_{2k-1}) = 4n - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(u''_{2k}) = 4mn + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v''_{2k-1}) = 5n - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v''_{2k}) = 4mn - n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

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$f(u^m{}_{2k-1}) = 2mn - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(u^m{}_{2k}) = 2mn + 4n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v^m{}_{2k-1}) = 2mn + n - 1 + 2k \qquad 1 \leq k \leq \frac{n}{2}$

$f(v^m{}_{2k}) = 2mn + 3n + 1 - 2k \qquad 1 \leq k \leq \frac{n}{2}$

Case:2 n is odd.

$f(u_{2k-1}) = 2k - 1 \qquad 1 \leq k \leq \frac{n+1}{2}$

$f(u_{2k}) = 4mn + 4n + 1 - 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_{2k-1}) = 4mn + 3n + 2 - 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v_{2k}) = n + 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(u'_{2k-1}) = 2n - 1 + 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(u'_{2k}) = 4mn + 2n + 1 - 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v'_{2k-1}) = 4mn + n + 2 - 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v'_{2k}) = 3n + 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(u''_{2k-1}) = 4n - 1 + 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(u''_{2k}) = 4mn + 1 - 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v''_{2k-1}) = 4mn - n + 2 - 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v''_{2k}) = 5n + 2k \qquad 1 \le k \le \frac{n-1}{2}$

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$f(u^m{}_{2k-1}) = 2mn - 1 + 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(u^m{}_{2k}) = 2mn + 4n + 1 - 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v^m{}_{2k-1}) = 2mn + 3n + 2 - 2k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v^m{}_{2k}) = 2mn + n + 2k \qquad 1 \le k \le \frac{n-1}{2}$

For this we define following edge function, $f^*: E(G) \to \{1,2,3, \dots 2mn + 2n - 1\}$ by

$f^*(u_k u_{k+1}) = 2mn + 2n - k \qquad 1 \le k \le n - 1$

$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 2mn + n$, n odd

$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 2mn + n$, n even

$f^*(v_k v_{k+1}) = 2mn + n - k \qquad 1 \le k \le n - 1$

$f^*(v_n u'_1) = 2mn$

$f^*(u'_k u'_{k+1}) = 2mn - k \qquad 1 \le k \le n - 1$

$f^*\left(u'_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right) = 2mn - n$, n odd

$f^*\left(u'_{\frac{n}{2}+1} v'_{\frac{n}{2}}\right) = 2mn - n$, n even

$f^*(v'_k v'_{k+1}) = 2mn - n - k \qquad 1 \le k \le n - 1$

$f^*(v'_n u''_1) = 2mn - 2n$

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$f^*(v^{m-1}{}_n u^m{}_1) = 2n$

$f^*(u^m{}_k u^m{}_{k+1}) = 2n - k$ $\quad 1 \le k \le n-1$

$f^*\left(u^m{}_{\frac{n+1}{2}} v^m{}_{\frac{n+1}{2}}\right) = n\,, n$ odd

$f^*\left(u^m{}_{\frac{n}{2}+1} v^m{}_{\frac{n}{2}}\right) = n\,, n$ even

$f^*(v^m{}_k v^m{}_{k+1}) = n - k$ $\quad 1 \le k \le n-1$

Thus the induced edge labels are distinct from $1,2,3,\ldots,2mn+2n-1$. Hence the graph m- Join of H_n is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of 2-Join of H_4 graph is shown in Figure-6.

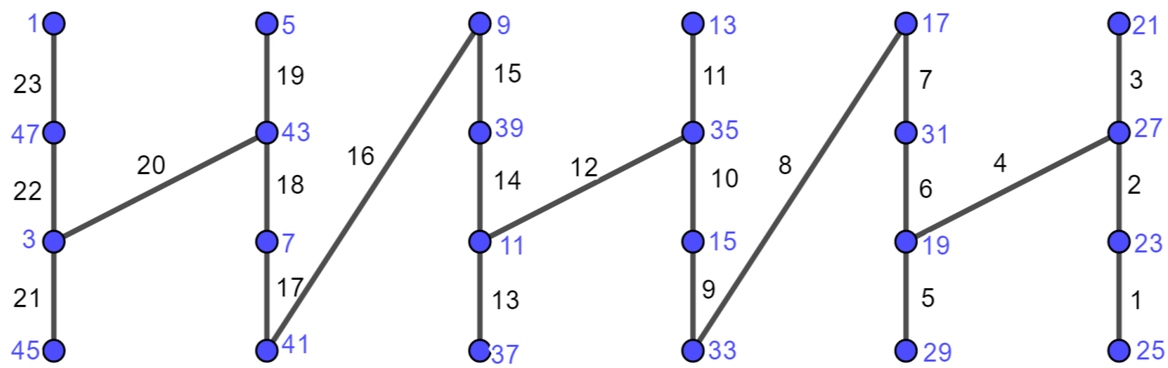

Figure-6 2-Join of H_4

Theorem 2.1.7 $P_n \odot K_{1,m}$ is an extra Skolem difference Mean graph

Proof: Let $G = P_n \odot K_{1,m}$

$V(G) = \{u_k, u'_k, u''_k, \ldots, u^m{}_k \;: 1 \le k \le n\}$

$E(G) = \{(u_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{(u_k\, u'_k): 1 \le k \le n\} \cup \{(u_k\, u''_k): 1 \le k \le n\} \cup \ldots \cup \{(u_k\, u^m{}_k): 1 \le k \le n\}$

So, $|V(G)| = n + nm$ & $|E(G)| = n + nm - 1$.

Define $f: V(G) \to \{1,2,3,\ldots,2n+2nm-1\}$.

$f(u_{2k-1}) = 2n + 2nm + 2m + 1 - (2m+2)k$ $\quad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$

$f(u_{2k}) = (2+2m)k - 1$ $\quad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(u'_{2k-1}) = (2+2m)k - (2m+1)$ $\quad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$

$f(u'_{2k}) = 2n + 2nm + 2m - 1 - (2m+2)k$ $\quad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(u''_{2k-1}) = (2+2m)k - (2m-1)$ $\quad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$

$f(u''_{2k}) = 2n + 2nm + 2m - 3 - (2m+2)k$ $\quad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

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$f(u^m{}_{2k-1}) = (2+2m)k - 3$ $\quad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$

$f(u^m{}_{2k}) = 2n + 2nm + 1 - (2m + 2)k \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

For this we define following edge function, $f^*: E(G) \to \{1,2,3, \ldots n + nm - 1\}$ by

$f^*(u_k u_{k+1}) = n + nm - (1 + m)k \qquad 1 \le k \le n - 1$

$f^*(u_k u'_k) = n + nm + m - (1 + m)k \qquad 1 \le k \le n$

$f^*(u_k u''_k) = n + nm + m - 1 - (1 + m)k \qquad 1 \le k \le n$

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$f^*(u_k u^m{}_k) = n + nm + 1 - (1 + m)k \qquad 1 \le k \le n$

Thus the induced edge labels are distinct from $1,2,3, \ldots, n + nm - 1$. Hence the graph $P_n \odot K_{1,m}$ is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of $P_4 \odot K_{1,3}$ is shown in Figure-7.

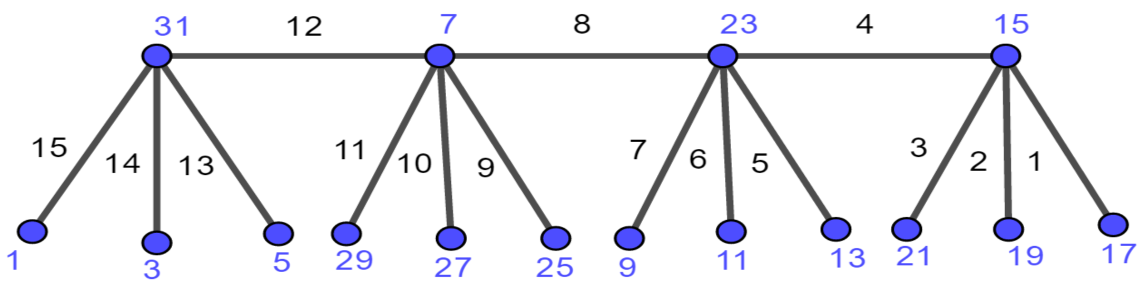

Figure-7 $P_4 \odot K_{1,3}$

Theorem 2.1.8 The $H-$ super subdivision of a path $HSS(P_n)$ is an extra Skolem difference mean graph.

Proof: Let $G = HSS(P_n)$

$V(G) = \{u_k, u^{(1)}_{k(k+1)}, u^{(1)}_{(k+1)k}, u^{(2)}_{k(k+1)}, u^{(2)}_{(k+1)k} : 1 \le k \le n - 1\} \cup \{u_n\}$

$E(G) = \{ u_k u^{(1)}_{k(k+1)}, u^{(1)}_{k(k+1)} u^{(2)}_{k(k+1)}, u^{(1)}_{k(k+1)} u^{(1)}_{(k+1)k}, u^{(1)}_{(k+1)k} u^{(2)}_{(k+1)k}, u_{k+1} u^{(1)}_{(k+1)k} : 1 \le k \le n - 1\}$

$So, |V(G)| = 5n - 4$ & $|E(G)| = 5n - 5$.

Define $f: V(G) \cup E(G) \to \{1,2,3, \ldots, 10n - 9\}$ as follows.

Case:1 n is even.

$f(u_{2k-1}) = 10k - 9 \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(u_{2k}) = 10n - 3 - 10k \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

$f\left(u^{(1)}_{(2k-1)(2k)}\right) = 10n + 1 - 10k \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

$f\left(u^{(1)}_{2k(2k-1)}\right) = 10k - 5 \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$

$f\left(u^{(1)}_{2k(2k+1)}\right) = 10k - 3 \qquad 1 \le k \le \left\lfloor \frac{n-1}{2} \right\rfloor$

$f\left(u^{(1)}_{(2k+1)(2k)}\right) = 10n - 7 - 10k \qquad 1 \le k \le \left\lfloor \frac{n-1}{2} \right\rfloor$

$f\left(u^{(2)}_{(2k-1)2k}\right) = 10k - 7 \qquad 1 \le k \le \left\lfloor\frac{n}{2}\right\rfloor$

$f\left(u^{(2)}_{(2k)(2k-1)}\right) = 10n - 1 - 10k \qquad 1 \le k \le \left\lfloor\frac{n}{2}\right\rfloor$

$f\left(u^{(2)}_{(2k+1)2k}\right) = 10k - 1 \qquad 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor$

$f\left(u^{(2)}_{(2k)(2k+1)}\right) = 10n - 5 - 10k \qquad 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor$

For this we define following edge function, $f^*: E(G) \to \{1,2,3, \ldots 5n-5\}$ as follows.

$f^*\left(u_k u^{(1)}_{k(k+1)}\right) = 5n - 5k \qquad 1 \le k \le n-1$

$f^*\left(u_{k+1} u^{(1)}_{(k+1)k}\right) = 5n - 4 - 5k \qquad 1 \le k \le n-2$

$f^*\left(u^{(1)}_{k(k+1)} u^{(1)}_{(k+1)k}\right) = 5n - 2 - 5k \qquad 1 \le k \le n-1$

$f^*\left(u^{(1)}_{k(k+1)} u^{(2)}_{k(k+1)}\right) = 5n - 1 - 5k \qquad 1 \le k \le n-1$

$f^*\left(u^{(1)}_{(k+1)k} u^{(2)}_{(k+1)k}\right) = 5n - 3 - 5k \qquad 1 \le k \le n-1$

$f^*\left(u_n u^{(1)}_{n(n-1)}\right) = 1$

Thus the induced edge labels are distinct from $1,2,3, \ldots, 5n-5$.

Hence the graph $HSS(P_n)$ is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of $HSS(P_4)$ is shown in Figure-8.

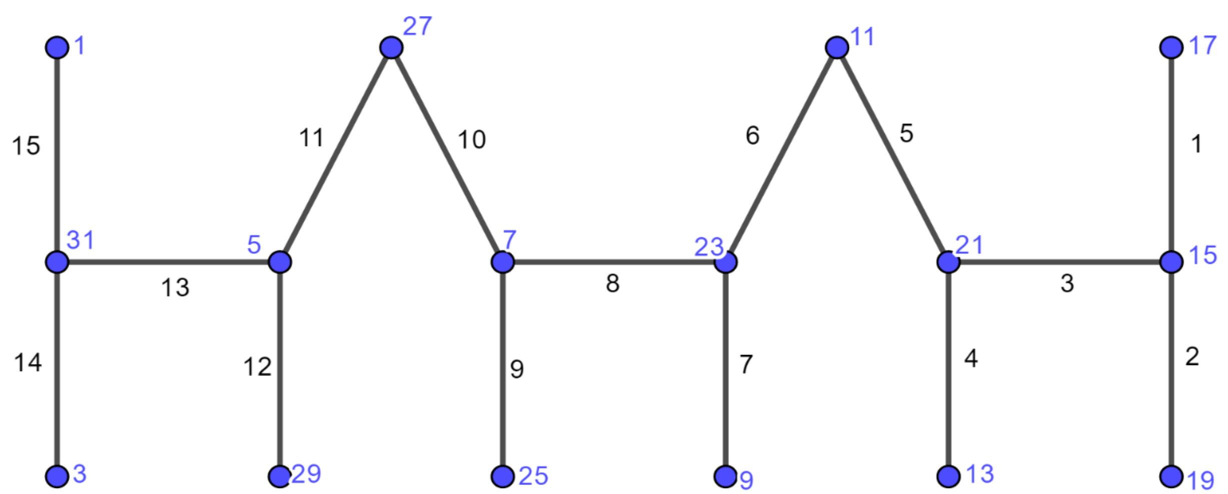

Figure-8 $\boldsymbol{HSS(P_4)}$

Theorem 2.1.9 $H \odot mK_1$ –graph of a path P_n is a Difference perfect square cordial graph.

Proof: Let $G = H \odot mK_1$ graph of a path P_n.
$V(G) = \{u_k, v_k, u_{ki}, v_{ki} : 1 \le k \le n, 1 \le i \le m\}$,
$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le n-1\} \cup \{u_k u_{ki} : 1 \le k \le n, 1 \le i \le m\} \cup \{v_k v_{ki} : 1 \le k \le n, 1 \le i \le m\} \cup \{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}} : n\ odd\}$ OR
$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le n-1\} \cup \{u_k u_{ki} : 1 \le k \le n, 1 \le i \le m\} \cup \{v_k v_{ki} : 1 \le k \le n, 1 \le i \le m\} \cup \{u_{\frac{n}{2}+1} v_{\frac{n}{2}} : n\ even\}$

So, $|V(G)| = 2n + 2nm$ & $|E(G)| = 2n + 2nm - 1$.
Define a function $f: V(G) \rightarrow \{1,2,3,\dots,4n + 4nm - 1\}$ as,
Case 1 : n is even.

$f(u_{2k-1}) = 4n + 4nm + 2m + 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(u_{2k}) = (2 + 2m)k - 1 \qquad 1 \le k \le \frac{n}{2}$
$f(u_{(2k-1)1}) = (2 + 2m)k - (2m + 1) \qquad 1 \le k \le \frac{n}{2}$
$f(u_{(2k-1)2}) = (2 + 2m)k - (2m - 1) \qquad 1 \le k \le \frac{n}{2}$
$f(u_{(2k-1)3}) = (2 + 2m)k - (2m - 3) \qquad 1 \le k \le \frac{n}{2}$
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$f(u_{(2k-1)m} = (2 + 2m)k - 3 \qquad 1 \le k \le \frac{n}{2}$
$f(u_{(2k)1} = 4n + 4nm + 2m - 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(u_{(2k)2}) = 4n + 4nm + 2m - 3 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(u_{(2k)3}) = 4n + 4nm + 2m - 5 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
.
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.
$f(u_{(2k)m}) = 4n + 4nm + 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{2k-1}) = 3n + 3nm + 2m + 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{2k}) = n + nm - 1 + (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{(2k-1)1}) = n + nm - 1 - 2m + (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{(2k-1)2}) = n + nm + 1 - 2m + (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{(2k-1)3}) = n + nm + 3 - 2m + (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
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.
$f(v_{(2k-1)m}) = n + nm - 3 + (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{(2k)1} = 3n + 3nm + 2m - 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{(2k)2} = 3n + 3nm + 2m - 3 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
$f(v_{(2k)3} = 3n + 3nm + 2m - 5 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$
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$f(v_{(2k)m} = 3n + 3nm + 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n}{2}$

Case 2 : n is odd.

$f(u_{2k-1}) = 4n + 4nm + 2m + 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n+1}{2}$
$f(u_{2k}) = (2 + 2m)k - 1 \qquad 1 \le k \le \frac{n-1}{2}$
$f(u_{(2k-1)1}) = (2 + 2m)k - (2m + 1) \qquad 1 \le k \le \frac{n+1}{2}$
$f(u_{(2k-1)2}) = (2 + 2m)k - (2m - 1) \qquad 1 \le k \le \frac{n+1}{2}$
$f(u_{(2k-1)3}) = (2 + 2m)k - (2m - 3) \qquad 1 \le k \le \frac{n+1}{2}$
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$f(u_{(2k-1)m} = (2 + 2m)k - 3 \qquad 1 \le k \le \frac{n+1}{2}$
$f(u_{(2k)1} = 4n + 4nm + 2m - 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$
$f(u_{(2k)2}) = 4n + 4nm + 2m - 3 - (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$
$f(u_{(2k)3}) = 4n + 4nm + 2m - 5 - (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$
.
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$f(u_{(2k)m}) = 4n + 4nm + 1 - (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_{2k-1}) = n + nm - m - 2 + (2 + 2m)k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v_{2k}) = 3n + 3nm + m - (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_{(2k-1)1}) = 3n + 3nm + 3m - (2 + 2m)k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v_{(2k-1)2}) = 3n + 3nm + 3m - 2 - (2 + 2m)k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v_{(2k-1)3}) = 3n + 3nm + 3m - 4 - (2 + 2m)k \qquad 1 \le k \le \frac{n+1}{2}$

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$f(v_{(2k-1)m}) = 3n + 3nm + m + 2 - (2 + 2m)k \qquad 1 \le k \le \frac{n+1}{2}$

$f(v_{(2k)1} = n - m + nm + (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_{(2k)2} = n - m + nm + 2 + (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_{(2k)3} = n - m + nm + 4 + (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$

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$f(v_{(2k)m} = n - m + nm + 2m - 2 + (2 + 2m)k \qquad 1 \le k \le \frac{n-1}{2}$

For this we define following edge function, $f^*: E(G) \to \{1,2,3, \dots 2n + 2mn - 1\}$ by

$f^*(u_k u_{k+1}) = 2n + 2nm - (m + 1)k \qquad 1 \le k \le n - 1$

$f^*(u_k u_{k1}) = 2n + 2nm + m - (m + 1)k \qquad 1 \le k \le n$

$f^*(u_k u_{k2}) = 2n + 2nm + m - 1 - (m + 1)k \qquad 1 \le k \le n$

$f^*(u_k u_{k3}) = 2n + 2nm + m - 2 - (m + 1)k \qquad 1 \le k \le n$

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$f^*(u_k u_{km}) = 2n + 2nm + 1 - (m + 1)k \qquad 1 \le k \le n$

$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = n + nm$, n even

$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = n + nm$, n odd

$f^*(v_k v_{k+1}) = n + nm - (m + 1)k \qquad 1 \le k \le n - 1$

$f^*(v_k v_{k1}) = n + nm + m - (m + 1)k \qquad 1 \le k \le n$

$f^*(v_k v_{k2}) = n + nm + m - 1 - (m + 1)k \qquad 1 \le k \le n$

$f^*(v_k v_{k3}) = n + nm + m - 2 - (m + 1)k \qquad 1 \le k \le n$

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$f^*(v_k v_{km}) = n + nm + 1 - (m + 1)k \qquad 1 \le k \le n$

Thus the induced edge labels are distinct from $1,2,3, \dots, 2n + 2nm - 1$.

Hence the graph $H \odot mK_1$ graph of a path P_n is an extra Skolem difference mean graph.

Illustration: An extra Skolem difference mean labeling of $H \odot 2K_1$ graph of a path P_5 is shown in Figure-9.

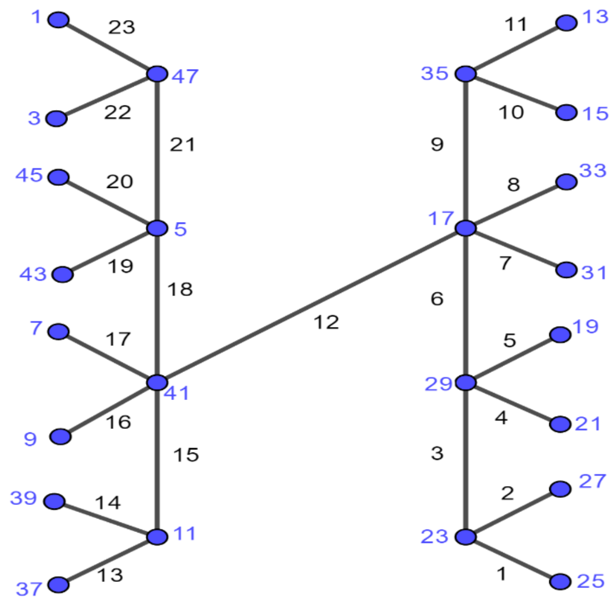

Figure-9 $H \odot 2K_1$ graph of a path P_5

3. Conclusion

In this paper we obtain an extra Skolem difference mean labeling for Comb graph, Twig of a path P_n, H graph of a path P_n, $K_{1,2} * K_{1,n}$ graph, $\mathrm{K}_{1,3} * \mathrm{K}_{1,\mathrm{n}}$ graph, $m-$ Join of H_n, $P_n \odot K_{1,m}$ graph , $HSS(P_n)$ graph, $H \odot mK_1$ −graph of a path P_n .We can discuss more similar results for various graphs.

References

[1] Gallian, J. A. (2018). A dynamic survey of graph labeling. *Electronic Journal of combinatorics*, *1*(DynamicSurveys), DS6.

[2] Gross, J. L., & Yellen, J. (2005). *Graph theory and its applications*. CRC press.

[3] Jeyanthi, P., & Gomathi, R. (2020). Analytic odd mean labeling of Corona graphs. *Malaya Journal of Matematik, 8*(3), 898-902

[4] Murugan, K., & Subramanian, A. (2012). Skolem difference mean graphs. *Mapana Journal of Sciences*, *11*(4), 109-120.

[5] Parmar, D., & Vaghela, U. (2019). Skolem difference mean labeling of some path related graphs. *Pramana Research Journal*, *9*(5), 651-657.

[6] Ramya, D., Selvi, M., & Kalaiyarasi, R. (2013). On skolem difference mean labeling of graphs. *International Journal of Mathematical Archive*, *4*(12), 73-79.

[7] Selvi, M., Ramya, D. & Jeyanthi, P. (2015). Skolem Difference mean graphs, Proyecciones. *Journal of Mathematics*, 34,243-254.

[8] Sundari, G. M., & Murugan, K. (2020). Extra Skolem Difference Mean Labeling of Some Graphs. *World Scientific News*, *145*, 210-221.

[9] Vaghela, U. V., & Parmar, D. B. (2020). Difference perfect square cordial labeling of some graphs. *Journal of Xidian University*, *14*(2).

[10] Sriram, S., & Govindarajan, R. (2019). Parity combination cordial labelling of joins of H graph, *International Journal of Research and Analytical Reviews* , 6, 28X-33X.