| RESEARCH ARTICLE

Orthogonal Jordan Triple Higher Reverse Left(resp.Right)Centralizer on Semiprime Γ-Rings

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| ABSTRACT

The goal of this job offer the entrance of orthogonal Jordan triple higher reverse left (resp.right) centralizers a $\cdot$-ring Ň (mention $\cdot$-ring Ň by Ň) & examine this concept

I - preamble:

implication of orthogonal Jordan triple higher reverse left (resp.right)centralizers (for short

o.j.t.h.r.l(resp.r)c is a substantive in non-commutative Algebra.See [1] & [2] show the defined. sp - Γr &

2-torsion free Γ-ring (for short 2- tf) resp. Ň, near at hand [2&3] .

Need the next Lemmas :

Lemma (1.1): [1]

If Ň is a 2-tf sp - Γr & χ , υ be elements of Ň, the next state are equivalent :

(i) $\chi\Gamma\zeta\Gamma\upsilon = 0$

(ii) $\upsilon\Gamma\zeta\Gamma\chi = 0$

(iii) $\chi\Gamma\zeta\Gamma\upsilon + \upsilon\Gamma\zeta\Gamma\chi = 0$, $\forall\ \zeta \in$ Ň

If one of these state is fulfilled ,then $\chi\Gamma\upsilon = \upsilon\Gamma\chi = 0$.

Lemma (1.2): [2]

If Ň is a 2-tf sp - Γr & χ , υ be elements of Ň if $\chi\Gamma\zeta\Gamma\upsilon + \upsilon\Gamma\zeta\Gamma\chi = 0$, $\forall\ \zeta \in$ Ň. Then $\chi\Gamma\zeta\Gamma\upsilon = \upsilon\Gamma\zeta\Gamma\chi = 0$.

II-Orthogonal Jordan triple higher reverse left (resp.right) centralizer on Γ-Rings:

Definition (2.1):

Two j.t.h.r.l(resp.r)c $\text{y} = (\text{y}_i)_{i \in N}$ & $\hbar = (\hbar_i)_{i \in N}$ of $\check{N}$ are named orthogonal if $\text{y}_n(\text{x})\Gamma\check{N}\ \Gamma\hbar_n(\text{v}) = (0) = \hbar_n(\text{v})\Gamma\check{N}\Gamma\text{y}_n(\text{x})$,

$\forall\ \text{x}, \text{v} \in \check{N}$ & $n \in N$.

Lemma (2.2):

Let $\check{N}$ be a 2-tf sp - Γr , $\text{y} = (\text{y}_i)_{i \in N}$ & $\hbar = (\hbar_i)_{i \in N}$ are two j.t.h.r.l(resp.r)c of $\check{N}$,where y_n , $\hbar_n$ are commuting .Then y_n & $\hbar_n$ are orthogonal iff $\text{y}_n(\text{x})\Gamma\check{N}\Gamma\ \hbar_n(\text{v}) + \hbar_n(\text{x})\Gamma M\Gamma\ \text{y}_n(\text{v}) = 0$,$\forall\ \text{x}, \text{v} \in \check{N}$ & $n \in N$.

Proof :

Supposition y_n & $\hbar_n$ are orthogonal

T.P. $\text{y}_n(\text{x})\Gamma\check{N}\Gamma\ \hbar_n(\text{v}) + \hbar_n(\text{x})\Gamma\check{N}\Gamma\ \text{y}_n(\text{v}) = 0$

Show y_n & $\hbar_n$ are orthogonal

Subsequently, according Lemma (1.1)

$$\sum_{i=1}^{n}\ \text{y}_i(\text{x})\ \alpha\ \hbar_i(\text{v})\ = 0 = \sum_{i=1}^{n}\ \hbar_i(\text{x})\ \alpha\ \text{y}_i(\text{v})$$

Right multiply according $\beta\ \text{y}_{i-1}(\text{z})$

$$\sum_{i=1}^{n}\ \text{y}_i(\text{x})\ \alpha\ \hbar_i(\text{v})\ \beta\ \text{y}_{i-1}(\text{z})\ + \sum_{i=1}^{n}\ \hbar_i(\text{x})\ \alpha\ \text{y}_i(\text{v})\ \beta\ \text{y}_{i-1}(\text{z})\ = 0$$

Show y_n & $\hbar_n$ are commuting , it come

Conversely, Supposition that $\text{y}_n(\text{x})\ \Gamma\ \check{N}\ \Gamma\ \hbar_n(\text{v}) + \hbar_n(\text{x})\ \Gamma\ \check{N}\ \Gamma\ \text{y}_n(\text{v}) = (0)$

T.P. y_n & $\hbar_n$ are orthogonal

According Lemma (1.2) , get it

Theorem (2.3):

Let $\check{N}$ be a 2-tf sp - Γr , $\text{y} = (\text{y}_i)_{i \in N}$ & $\hbar = (\hbar_i)_{i \in N}$ are two j.t.h.r.l(resp.r)c of $\check{N}$,where y_n , $\hbar_n$ are commuting. Then the next state are equivalent, every $n \in N$:

(1) y_n & h_n are orthogonal

(2) $\text{y}_n\hbar_n = 0$

(3) $\hbar_n\text{y}_n = 0$

(4) $\text{y}_n\hbar_n\ + \hbar_n\text{y}_n = 0$

Proof : (1) $\Leftrightarrow$ (2)

Supposition y_n & $\hbar_n$ are orthogonal

T.P. $\text{y}_n\hbar_n = 0$

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Show $ɏ_n$ & $ɏ_n$ are orthogonal

According Lemma (1.1)

$$\sum_{i=1}^{n} ɏ_i(ħ_i(ѵ)) \ \alpha \ ɏ_{i-1}(ɏ_i(ӿ)) = 0$$

Right multiply according $\beta \ ɏ_i(ħ_i(ѵ))$

$$\sum_{i=1}^{n} ɏ_i(ħ_i(ѵ)) \ \alpha \ ɏ_{i-1}(ɏ_i(ӿ)) \ \beta \ ɏ_i(ħ_i(ѵ)) = 0$$

Show Ň is a sp - Γr , it come

Conversely , supposition $ɏ_n ħ_n = 0$

T.P. $ɏ_n$ & $ħ_n$ are orthogonal

$$\sum_{i=1}^{n} ɏ_i(ħ_{i-1}(ѵ)) \ \alpha \ ɏ_{i-1}(ħ_{i-1}(ẕ)) \ \beta \ ɏ_{i-1}(ħ_i(ӿ)) = 0$$

Replace $ħ_{i-1}(ѵ)$ According $ӿ$ & $ɏ_{i-1}(ħ_{i-1}(ӿ))$ According $ħ_i(y)$

$$\sum_{i=1}^{n} ɏ_i(ӿ) \ \alpha \ ɏ_{i-1}(ħ_{i-1}(ẕ)) \ \beta \ ħ_i(ѵ) = 0 \quad ... (1)$$

Show $ɏ_n$ & $ħ_n$ are commuting

$$\sum_{i=1}^{n} ħ_i(ѵ) \ \alpha \ ɏ_{i-1}(ħ_{i-1}(ẕ)) \ \beta \ ɏ_i(ӿ) = 0 \quad ... (2)$$

According (1) & (2) , get it

Proof : $(1) \Leftrightarrow (3)$

According same in $(1) \Leftrightarrow (2)$, get it

Proof : $(1) \Leftrightarrow (4)$

Supposition $ɏ_n$ & $ħ_n$ are orthogonal

By (ii) & (iii) , get

Conversely , Supposition $ɏ_n ħ_n + ħ_n ɏ_n = 0$

T.P. $ɏ_n$ & $ħ_n$ are orthogonal

$$\sum_{i=1}^{n} ɏ_i(ħ_{i-1}(ѵ)) \ \alpha \ ɏ_{i-1}(ħ_{i-1}(ẕ)) \ \beta \ ɏ_{i-1}(ħ_i(ӿ)) + ħ_i(ɏ_{i-1}(ѵ)) \ \alpha \ ħ_{i-1}(ɏ_{i-1}(ẕ)) \ \beta \ ħ_{i-1}(ɏ_i(ӿ)) = 0$$

Replace $ħ_{i-1}(ѵ)$ by $ӿ$, $ɏ_{i-1}(ѵ)$ by $ӿ$, $ɏ_{i-1}(ħ_i(ӿ))$ by $ħ_i(ѵ)$ & $ħ_{i-1}(ɏ_i(ӿ))$ by $ɏ_i(ѵ)$

$$\sum_{i=1}^{n} ɏ_i(ӿ) \ \alpha \ ɏ_{i-1}(ħ_{i-1}(ẕ)) \ \beta \ ħ_i(ѵ) + ħ_i(ӿ) \ \alpha \ ħ_{i-1}(ɏ_{i-1}(ẕ)) \ \beta \ ɏ_i(ѵ) = 0$$

According Lemma (2.2) , get it

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Theorem(2.4):

Let Ň be a 2-tf sp - Γr, ɏ=(ɏ$_i$)$_{i∈N}$ & ħ=(ħ$_i$)$_{i∈N}$ be two j.t.h.r.l(resp.r)c ,where ɏ$_n$, ħ$_n$ are commuting.Then the next state are equivalent ,∀ x, v, ∈ Ň & n ∈ N :

(a) ɏ$_n$ & h$_n$ are orthogonal

(b) ɏ$_n$(x) Γ ħ$_n$(v) = 0

(c) ħ$_n$(x) Γ ɏ$_n$(v) = 0

(d) ɏ$_n$(x) Γ ħ$_n$(v) + ħ$_n$(x) Γ ɏ$_n$(v) = 0

Proof : (a) ⇔ (b)

Supposition ɏ$_n$ & ħ$_n$ are orthogonal

T.P. ɏ$_n$(x) Γ ħ$_n$(v) = 0

Show ɏ$_n$ & ħ$_n$ are orthogonal , according Lemma (1.1) , get it

Conversely, supposition ɏ$_n$(x) Γ ħ$_n$(v) = (0)

T.P. ɏ$_n$ & ħ$_n$ are orthogonal

Right multiply according β ɏ$_{i-1}$ (z)

$$\sum_{i=1}^{n} ɏ_i(x) \; α \; ħ_i(v) \; β \; ɏ_{i-1}(z) = 0$$

Show ɏ$_n$ & ħ$_n$ are commuting

$$\sum_{i=1}^{n} ɏ_i(x) \; α \; ɏ_{i-1}(z) \; β \; ħ_i(v) = 0 \qquad ...(1)$$

Show ɏ$_n$ & ħ$_n$ are commuting

$$\sum_{i=1}^{n} ħ_i(v) \; α \; ɏ_{i-1}(z) \; β \; ɏ_i(x) = 0 \qquad ...(2)$$

According (1) & (2) , get this

Proof : (a) ⇔ (c)

According same in (a) ⇔ (b) , get (a) ⇔ (c) .

Proof : (a) ⇔ (d)

Supposition ɏ$_n$ & ħ$_n$ are orthogonal

According (b) & (c), get it

Conversely, Supposition ɏ$_n$(x) Γ ħ$_n$(v) + ħ$_n$(x) Γ ɏ$_n$(v) = (0)

T.P. ɏ$_n$ & ħ$_n$ are orthogonal

$$\sum_{i=1}^{n} ɏ_i(x) \; α \; ħ_i(v) + \sum_{i=1}^{n} ħ_i(x) \; α \; ɏ_i(v) = 0$$

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Right multiply according $\beta\ y_{i-1}(z)$

$$\sum_{i=1}^{n} y_i(x)\ \alpha\ \hbar_i(y)\ \beta\ y_{i-1}(z)\ +\ \sum_{i=1}^{n}\ \hbar_i(x)\ \alpha\ y_i(y)\ \beta\ y_{i-1}(z) = 0$$

Show y_n , $\hbar_n$ are commuting & According Lemma (2.2) , get it .

Theorem (2.5):

Let $\check{N}$ be a 2-tf sp - Γr, $y = (y_i)_{i \in N}$ & $\hbar = (\hbar_i)_{i \in N}$ are two j.t.h.r.l(resp.r)c of $\check{N}$, Supposition $y_n\ \alpha\ y_n = \hbar_n\ \alpha\ \hbar_n$. Then $y_n + \hbar_n$ & $y_n - \hbar_n$ are orthogonal .

Proof :

$((y_n + \hbar_n)\ \alpha\ (y_n - \hbar_n) + (y_n - \hbar_n)\ \alpha\ (y_n + \hbar_n)\ )\ (x)$

$$= \sum_{i=1}^{n}\ y_n\ \alpha\ y_n - y_i(x)\ \alpha\ \hbar_i(x) + \hbar_i(x)\ \alpha\ y_i(x) - \hbar_n\ \alpha\ \hbar_n + y_n\ \alpha\ y_n + y_i(x)\ \alpha\ \hbar_i(x) - \hbar_i(x)\ \alpha\ y_i(x) - \hbar_n\ \alpha\ \hbar_n$$

$= 0$

Subsequently , $((y_n + \hbar_n)\ \alpha\ (y_n - \hbar_n) + (y_n - \hbar_n)\ \alpha\ (y_n + \hbar_n))(x) = 0$

According Theorem (2.4)(d) $\Rightarrow$ (a) , get it

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