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| RESEARCH ARTICLE

A Least Squares Hyperbolic Transformation to Moderate School Assessments

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| ABSTRACT

This research develops a new method to mathematically moderate school assessments to make them comparable across schools in a large-scale public examination system. In many systems the school assessment is partly used to determine the final mark in a course. A hyperbola was chosen for this method as it has two important features. First, if chosen from the second or fourth quadrants of the Cartesian plane it is always monotonic increasing. Second, it gives a curvilinear transformation which helps to overcome differences in skew between a school's raw assessments and the school's exam marks. If the skew difference is large it can disadvantage the top students in a linear moderation, giving them moderated assessments below their exam marks. A hyperbola is closely fitted to the moderation criterion (the sorted exam marks) using a least squares process. The moderation equation itself is simple, with three constants, but their calculation is more complex. However a central education authority should have the computing resources to calculate these constants for each school group. This new method provides another option for school systems to consider in a high-stakes large-scale public examinations environment.

| KEYWORDS

school assessments, moderation method, least squares, external examinations, high stakes, large scale.

| ARTICLE INFORMATION

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1. Introduction

In many public examination systems, the final award is partly based on school assessments. These are generally based on a series of school-based assessment tasks that are set and marked locally at the school. Instead of a 'one-shot' method of testing, these tasks are spread throughout the final school year. For each school course (e.g. Physics at school X), the marks for such assessment tasks are aggregated at the school level to form a final raw assessment mark for each pupil which is then submitted to the central education authority. At the Higher School Certificate (HSC) in NSW, Australia, the moderated assessment counts equally with the public exam result in determining the Australian Tertiary Admission Rank (ATAR) and comes under considerable scrutiny.

The raw assessments submitted by different schools cannot be assumed to be comparable, as schools can differ greatly in the extent to which their standards adhere to a common scale (Willingham, 1963). The process of adjusting the assessments so that they do conform to a common scale is known as moderation. Two types of moderation may be distinguished. The first using statistical procedures and a criterion to adjust the assessments is referred to as 'statistical moderation'. The second, a procedure that involves human judgements to attempt assessment parity, is referred to as 'social moderation' (e.g. Mislevy, 1992; Linn, 1996). Prichard et al. (2025) and Chambers et al. (2024) focus extensively on non-statistical moderation but with these methods it is difficult to get an acceptable level of comparability for large-scale systems. Comprehensive surveys of statistical moderation have been made by Wilmut and Tuson (2005) and Williamson (2016). This paper will will develop a conceptually simple method of statistical moderation for both linear and non linear conversions that smoothly adjusts a set of assessment marks so that they closely approximate a set of criterion scores.

2. Literature Review

2.1 The Linear Model

In 1977, when assessments were introduced as part of the HSC award, the moderation method was linear. This linear adjustment, involving the mean and standard deviation, has a long history. It was originally devised by Francis Galton in the late nineteenth century, put into its current form by Karl Pearson around the turn of the twentieth century, and used in an educational context in Britain by Cyril Burt before the First World War (Howard, 1958). It has been used in Britain and in countries with educational systems modelled on the British (Pilliner, 1958; Lubisi and Murphy, 2002; Hong Kong Examinations and Assessment Authority, 2023). In its simplest form the assessments are adjusted to have the same mean and standard deviation as the exam marks as follows:

$$M = \mu_e + \frac{\sigma_e}{\sigma_x}(x - \mu_x) \qquad (1)$$

where *x* is a pupil's raw school assessment, *M* is the pupil's moderated assessment, μ_x and μ_e are the school group means for the assessments (*x*) and examination marks (*e*), and σ_x and σ_e are the respective standard deviations.

Statistical moderation requires a criterion, in this case the examination marks scored by the school group. The key idea is that the assessments distribution must be made to resemble the exam marks distribution, except for the rank orders. For a linear conversion, the usual way of doing this is to match the means and standard deviations (Linn, 1966). An implicit assumption of the linear method is that the school group assessments and the examination marks have similar distribution shapes. This assumption often does not hold. Quite commonly the assessments are negatively skewed, with students bunched near the top of the mark range and tailing off at the lower mark levels. The external examination marks may spread the students out more effectively, with this distribution being more symmetrical. The result of applying a linear moderation to this situation is that the top moderated assessments can fall short of the top examination marks as the bunching creates smaller assessment *z* (standard) scores at the top than for the exam marks. The opposite case involves positively skewed assessments where one or two students are placed well ahead of the rest of the group, but on the examination, marks are somewhere in the pack or only just above the others. This situation can give moderated assessments that exceed the top examination marks scored by the group. In some cases, this effect could result in the moderated assessment exceeding the top of the mark scale requiring it to be fitted into the mark range.

2.2. Polynomial Models

Where the two distributions have significantly different skews the linear moderation process can be problematic. In such cases a transformation which equates percentiles, giving a curved line of adjustment, would be appropriate if the school groups were sufficiently large (Angoff, 1971; Kolen and Brennan, 2014). However, most school groups are not large enough to effectively use equipercentile procedures. The principle adopted by psychometricians is that if the distributions differ significantly in their shapes then a curved line of relationship should be used. At the 1993 HSC this led to a replacement of the linear method with a moderation that used a quadratic polynomial, as follows:

$$M = ax^2 + bx + c \qquad (2)$$

where *x* is a pupil's raw assessment, *M* is the pupil's moderated assessment, and *a*, *b*, *c* are constants for each school group.

This method is an improvement over the linear method for curvilinear relationships. However in practice, it often needs to be modified because of problems in gaining a monotonic increasing curve – a curve where the moderated assessments increase when the raw assessments increase. Such problems also occur for higher order polynomials.

3. Methodology

3.1 The Hyperbolic Model

The hyperbola is chosen to be always monotonic increasing by applying the appropriate constraints. The practical importance of this is considerable. If a quadratic or a cubic is fitted by some process, the curves can arrange themselves to best fit the data by creating a maximum or minimum value in the assessment mark range. Even if they are monotonic increasing within the assessment mark range, they may become non monotonic if an extrapolation is required. The hyperbolic moderation equation is given by

$$M = \frac{c}{x+a} - b \qquad (3)$$

where *x* is a student's raw assessment, *M* is the moderated assessment and *a*, *b* and *c* are constants to be determined for each school group, where $a \neq -x$ for all assessments.

The choice of a Least Squares method focuses our attention on the distribution of the criterion. Under the traditional linear model, summary statistics like the mean and standard deviation were of interest. Under the Least Squares method, the distribution of the criterion is of interest. Here, the criterion scores (*y*) are simply the school group exam marks (*e*) sorted in the same order as the raw assessments. That is, the top exam mark is associated with the top raw assessment, the second top exam mark is associated with the second top raw assessment, and so on. If there are tied scores on the raw assessments (*x*), then the corresponding *y* scores are averaged. For the HSC exams, the raw assessments are integers ranging from 1 to 50.

When the *y* scores are graphed against the *x* scores, the curve is not smooth but often somewhat ragged. The ragged features of this curve are assumed to not represent meaningful features of the graph. A hyperbola is then fitted to the graphed points so as to minimize the sum of the squared vertical deviations. This Least Squares method ensures that the resulting hyperbolic curve smoothly approximates the plot of *y* against *x*, fitting it closely. The constraints on the *a* constant for a Concave Up and Concave Down hyperbola are derived below.

3.2 Restrictions on constant c

For a monotonic increasing curve, the first derivative of (3) must be positive.

$$\frac{dM}{dx} = \frac{-c}{(x+a)^2} \quad (4)$$

$$\therefore c < 0 \quad (5)$$

3.3 Restrictions on constant a

For a Concave Up Hyperbola:

$$\frac{d^2M}{dx^2} > 0 \quad (6)$$

$$\frac{d^2M}{dx^2} = \frac{2c}{(x+a)^3} \quad (7)$$

Therefore from (5), (6), (7) and as $x > 0$

$$x + a < 0$$

$$\therefore a < -x_{max} \quad (8)$$

For a Concave Down Hyperbola:

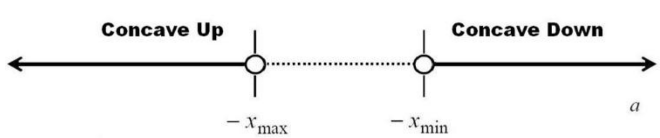
$$\frac{d^2M}{dx^2} < 0 \quad (9)$$

Therefore from (5), (7), (9) and as $x > 0$

$$x + a > 0$$

$$\therefore a > -x_{min} \quad (10)$$

From (8) and (10) the values of *a* are restricted as follows:



4. The Least Squares Curve Fitting

The vertical distance between each datum point (*x*, *y*) and each corresponding point on the fitted hyperbola (*x*, *M*) is squared and summed over all data points for the students in a particular course and school to give a value *L*. The Least Squares solution minimizes this value. The quantity to be minimized is given by the following where the summation is over the *n* students in a particular school/course group.

$$L = \sum_{i=1}^{n} [y_i - M_i]^2$$

Substituting from equation (3) gives

$$L = \sum_{i=1}^{n} \left[y_i - \left(\frac{c}{x_i + a} - b\right)\right]^2 \tag{11}$$

Expanding (11) gives

$$L = \sum_i {y_i}^2 - 2c \sum_i \frac{y_i}{(x_i + a)} + 2b \sum_i y_i + c^2 \sum_i \frac{1}{(x_i + a)^2} - 2bc \sum_i \frac{1}{(x_i + a)} + nb^2 \tag{12}$$

To find the values of *a*, *b* and *c* that minimize *L*, equation 12 is partially differentiated with respect to each of these three parameters. The partial derivatives are then each set to zero to obtain a minimum value of the function. (Maximum values are inappropriate to this situation). The three equations obtained are then solved for *a*, *b* and *c*. This technique of setting the partial derivatives to zero to find the values that minimize or maximize a function has long been used in psychometrics (for example, Lord, 1955).

4.1 The Partial Derivative of L with respect to a

$$\frac{\partial L}{\partial a} = 2c \sum_i \frac{y_i}{(x_i + a)^2} - 2c^2 \sum_i \frac{1}{(x_i + a)^3} + 2bc \sum_i \frac{1}{(x_i + a)^2} \tag{13}$$

As $c \neq 0$, and setting (13) equal to zero, this gives

$$\sum_i \frac{y_i}{(x_i + a)^2} + b \sum_i \frac{1}{(x_i + a)^2} - c \sum_i \frac{1}{(x_i + a)^3} = 0 \tag{14}$$

4.2 The Partial Derivative of L with respect to b

$$\frac{\partial L}{\partial b} = 2 \sum_i y_i - 2c \sum_i \frac{1}{(x_i + a)} + 2nb \tag{15}$$

Setting (15) equal to zero gives

$$b = \frac{c}{n} \sum_i \frac{1}{(x_i + a)} - \bar{y} \tag{16}$$

4.3 The Partial Derivative of L with respect to c

$$\frac{\partial L}{\partial c} = -2 \sum_i \frac{y_i}{(x_i + a)} + 2c \sum_i \frac{1}{(x_i + a)^2} - 2b \sum_i \frac{1}{(x_i + a)} \tag{17}$$

Setting (17) equal to zero gives

$$c \sum_i \frac{1}{(x_i + a)^2} - \sum_i \frac{y_i}{(x_i + a)} - b \sum_i \frac{1}{(x_i + a)} = 0 \tag{18}$$

4.4. Solving for the Constants

Eliminating *b* from (18) by substituting from (16) gives

$$c\sum_i \frac{1}{(x_i+a)^2} - \sum_i \frac{y_i}{(x_i+a)} - \left[\frac{c}{n}\sum_i \frac{1}{(x_i+a)} - \bar{y}\right]\sum_i \frac{1}{(x_i+a)} = 0$$

Expanding and solving for *c* in terms of *a* gives

$$c = \frac{\sum_i \frac{(y_i-\bar{y})}{(x_i+a)}}{\sum_i \frac{1}{(x_i+a)^2} - \frac{1}{n}\left(\sum_i \frac{1}{(x_i+a)}\right)^2} \tag{19}$$

Eliminating *b* from (14) by substituting from (16) gives

$$\sum_i \frac{y_i}{(x_i+a)^2} + \left[\frac{c}{n}\sum_i \frac{1}{(x_i+a)} - \bar{y}\right]\sum_i \frac{1}{(x_i+a)^2} - c\sum_i \frac{1}{(x_i+a)^3} = 0 \tag{20}$$

Expanding (20) gives

$$\frac{c}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^2} - c\sum \frac{1}{(x_i+a)^3} + \sum_i \frac{(y_i-\bar{y})}{(x_i+a)^2} = 0 \tag{21}$$

Solving for *c* gives

$$c = \frac{\left[\sum_i \frac{(y_i-\bar{y})}{(x_i+a)^2}\right]}{\left[\sum_i \frac{1}{(x_i+a)^3} - \frac{1}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^2}\right]} \tag{22}$$

Eliminating *c* from (19) and (22) gives (23), an equation solely in terms of the *a* parameter.

$$\left[\sum_i \frac{(y_i-\bar{y})}{(x_i+a)}\right]\left[\sum_i \frac{1}{(x_i+a)^3} - \frac{1}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^2}\right] - \left[\sum_i \frac{(y_i-\bar{y})}{(x_i+a)^2}\right]\left[\sum_i \frac{1}{(x_i+a)^2} - \frac{1}{n}\left(\sum_i \frac{1}{(x_i+a)}\right)^2\right] = 0 \tag{23}$$

The key to solving these equations is to find *a* from (23). Then *c* is obtained from (19) and *b* from (16). An iterative process is used to find the value of *a* in Equation (23). This equation may be written as $F(a) = 0$.

4.5. What does F(a) look like?

Where a root exists, the general characteristics of *F*(*a*) are sketched below in Figure 1 for Concave Up and Figure 2 for Concave Down. In Concave Up, *F*(*a*) approaches +∞ at $-x_{max}$. After crossing below the *a*-axis it reaches a minimum and then approaches 0 as $a \to -\infty$.

The existence of a root is confirmed by constructing an interval and evaluating *F*(*a*) at the endpoints to see if it changes sign. The Bisecting the Interval method is also used to get an initial estimate of the root. The right side of the interval is set at a suitable value close to the left of $-x_{max}$ and the left side of the interval is set at $-a_c$, a critical value set at a large negative value. If the root is to the left of $-a_c$, it means that the curved hyperbola is approximating a near straight line and so a linear moderation can be used instead of the hyperbola.

For Concave Down, *F*(*a*) approaches −∞ at the $-x_{min}$ discontinuity. Here, *F*(*a*) crosses above the *a*-axis, reaches a maximum and then approaches 0 as $a \to \infty$.

The left side of the interval is set close to the right of $-x_{min}$ and the right side is set at a_c, a very large critical value. If the root is to the right of a_c, the hyperbola is approximating a near straight line and a linear moderation can be used.

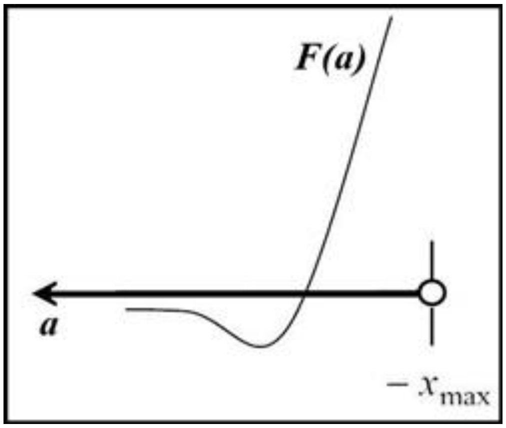


Figure 1: *F*(*a*) for Concave Up

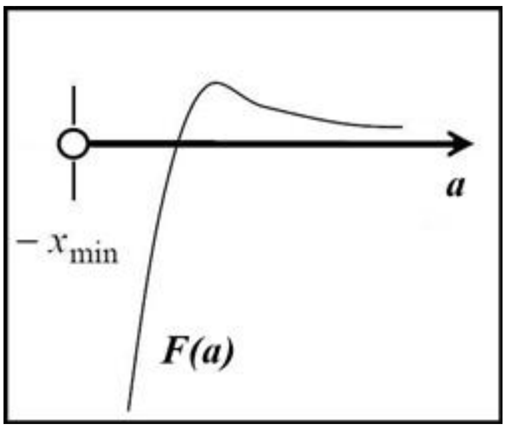


Figure 2: *F*(*a*) for Concave Down

A decision is needed on the size of a_c, a boundary governing what type of moderation is used. If the root is outside this boundary, a linear moderation is preferred; if inside this boundary, a hyperbolic moderation is used. Here the size of a_c is based on the experience from observing a number of cases. As many school group samples are small a statistical method of determining linearity may lack sufficient power. In NSW, the moderated assessments are out of 50 and are calculated to 1 decimal place.

Here a_c was set at ten times the range at 500 which seems conservative but may not be optimal. Other educational systems may prefer a different value if this method is implemented. Further research could be useful on this point. However, if a_c is set too small then a root for a genuine curved relationship may lie outside the boundary.

5. Applying the Newton-Raphson Method

The Newton-Raphson method requires that an initial estimate a_0 be made. The existence of a root is determined from the Bisecting the Interval method, which also gives a starting estimate for the root, a_0, after a few iterations. To avoid complications that may be caused by the maximum and minimum points, for the Concave Up case, the last value of *a* that made *F*(*a*) positive is taken as a_0 and for the Concave Down case, the last value of *a* that made *F*(*a*) negative is taken as a_0.

Then given a_0, a better estimate is given by a_1 as follows:

$$a_1 = a_0 - \frac{F(a_0)}{F'(a_0)} \tag{25}$$

a_1 is then the next estimate in (25) which generates a_2, and so on as the estimate converges.

5.1 An expression for $F'(a)$

Equation (23) may be written in terms of its products as

$$F(a) = G_1(a)G_2(a) - G_3(a)G_4(a), \text{ where} \tag{26}$$

$$G_1(a) = \sum_i \frac{(y_i - \bar{y})}{(x_i + a)} \tag{27}$$

$$G_2(a) = \sum_i \frac{1}{(x_i+a)^3} - \frac{1}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^2} \tag{28}$$

$$G_3(a) = \sum_i \frac{y_i - \bar{y}}{(x_i+a)^2} \tag{29}$$

$$G_4(a) = \sum_i \frac{1}{(x_i+a)^2} - \frac{1}{n}\left(\sum_i \frac{1}{(x_i+a)}\right)^2 \tag{30}$$

Then by the formula for differentiating a product:

$$F'(a) = \left[G_1 \frac{\partial G_2}{\partial a} + G_2 \frac{\partial G_1}{\partial a}\right] - \left[G_3 \frac{\partial G_4}{\partial a} + G_4 \frac{\partial G_3}{\partial a}\right] \tag{31}$$

Taking the partial derivatives of (27) to (30) with respect to *a* we obtain:

$$\frac{\partial G_1}{\partial a} = -\sum_i \frac{(y_i - \bar{y})}{(x_i+a)^2} \tag{32}$$

$$\frac{\partial G_2}{\partial a} = -3\sum_i \frac{1}{(x_i+a)^4} + \frac{2}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^3} + \frac{1}{n}\left[\sum_i \frac{1}{(x_i+a)^2}\right]^2 \tag{33}$$

$$\frac{\partial G_3}{\partial a} = -2\sum_i \frac{(y_i - \bar{y})}{(x_i+a)^3} \tag{34}$$

$$\frac{\partial G_4}{\partial a} = -2\sum_i \frac{1}{(x_i+a)^3} + \frac{2}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^2} \tag{35}$$

Substituting (27) to (30) and (32) to (35) into (31), and simplifying, we obtain:

$$\begin{aligned} F'(a) = &\left[\sum_i \frac{(y_i - \bar{y})}{(x_i+a)}\right]\left[-3\sum_i \frac{1}{(x_i+a)^4} + \frac{2}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^3} + \frac{1}{n}\left[\sum_i \frac{1}{(x_i+a)^2}\right]^2\right] \\ &+ \left[\sum_i \frac{(y_i - \bar{y})}{(x_i+a)^2}\right]\left[\sum_i \frac{1}{(x_i+a)^3} - \frac{1}{n}\sum_i \frac{1}{(x_i+a)}\sum_i \frac{1}{(x_i+a)^2}\right] \\ &+ \left[2\sum_i \frac{(y_i - \bar{y})}{(x_i+a)^3}\right]\left[\sum_i \frac{1}{(x_i+a)^2} - \frac{1}{n}\left(\sum_i \frac{1}{(x_i+a)}\right)^2\right] \end{aligned} \tag{36}$$

6. Small School/Course groups

A hyperbola obviously has no solution if the school group is less than 3. If there are 3 students, then it will give a solution if the 3 values for *x* are distinct and the 3 values for *y* are distinct. A formula can easily be worked out for this. However the curve obtained can hardly be trusted. The best solution is to combine the assessment programs for schools in the same region to build up the number under the supervision of a local assessment officer appointed by the central authority.

If this cannot be done and the school group number is 5 or less it is probably safest to initially use a simple linear moderation and have these school groups flagged. Then each school group results would need to be inspected by officers of the central authority to look for any anomalies.

A third line of validation would come from School Appeals against the moderated assessment results which would result in a further analysis and a possible change with an explanation to the school.

7. Examples of the Moderated Assessments (*M*)

The *M* values (red circles) were calculated for real school groups and worked exactly as expected. They closely follow the trend of the criterion (*y*) shown by open circles. Four examples from different courses are given. Here the *M* points are only plotted where a raw assessment exists although Equation (3) gives a continuous line. When pupils with suspect exam marks (e.g. illness cases) are removed from the process to establish the conversion line, their *M* values can be easily interpolated from Equation (3).

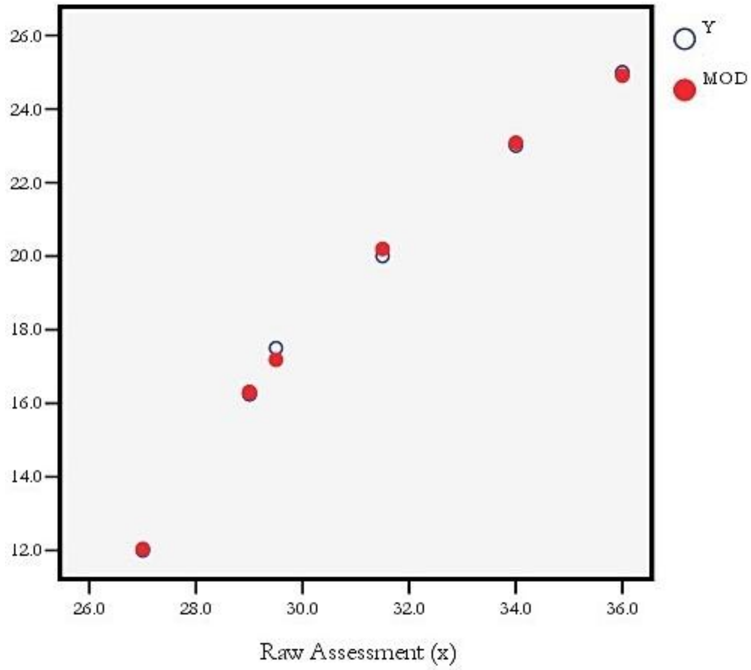


Figure 3 Course: Physics (n = 7)

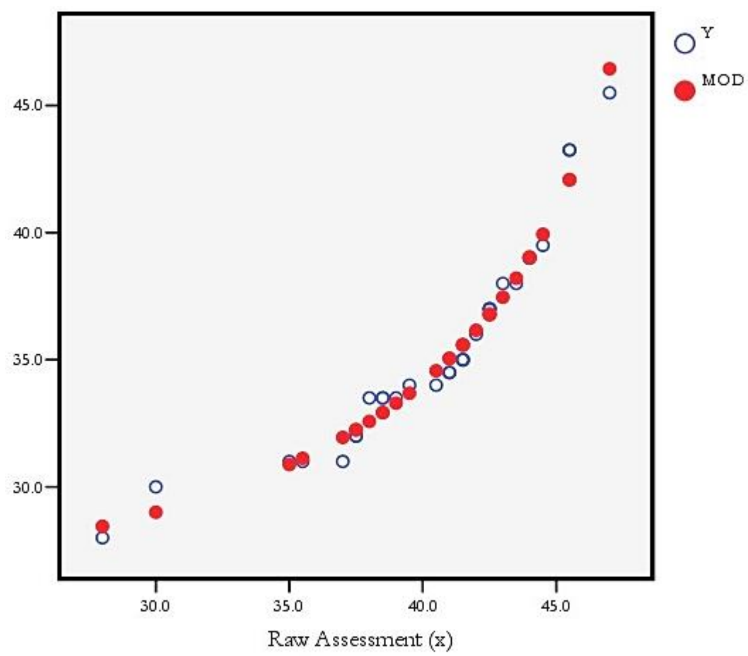


Figure 4 Course: Drama (n = 30)

The Drama raw assessments are more closely bunched at the top of the mark scale and more widely spaced near the bottom than the criterion scores, giving a strongly curved concave up moderation conversion line.

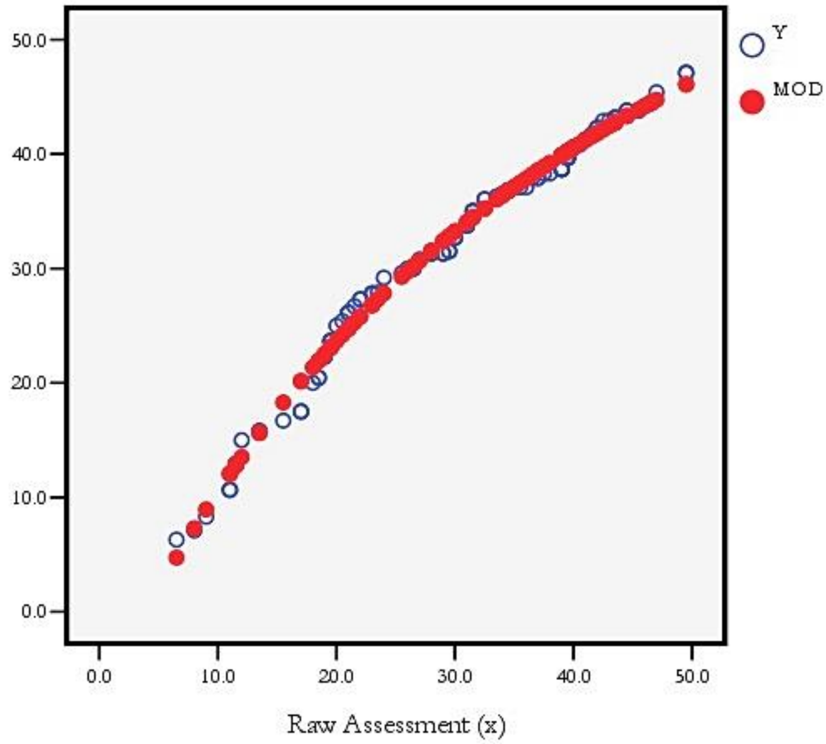


Figure 5 Course: Mathematics (n = 102)

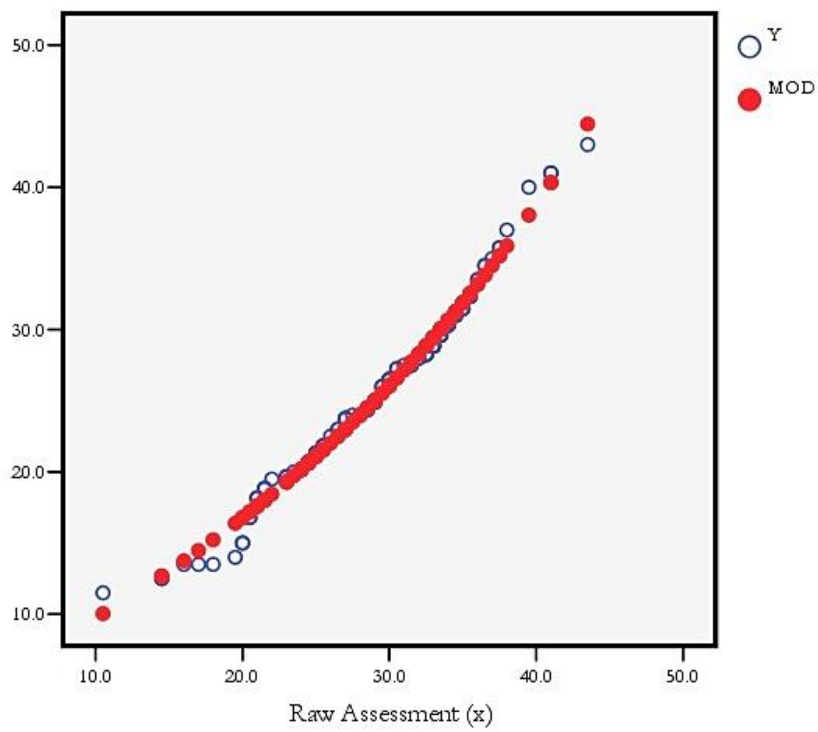


Figure 6 Course: English (n = 117)

Here the conversion line is concave up. There is a small group of low ability students who appear to have slightly underperformed on the examination.

8. Conclusion

The Least Squares method ensures that the moderation curve will closely fit the plot of the criterion scores but of course does not solve all the problems that can crop up in moderation. In some cases there may be outliers and moderation systems usually have additional techniques for identifying these. There are also Illness/misadventure cases whose exam marks may be suspect. These groups would be temporarily removed from the moderation process when the parameters of the moderation conversion line are calculated. The moderation would then be performed on all students, including those temporarily excluded. This technique can also be applied to the hyperbolic moderation.

All moderation systems, regardless of the method used, also have to consider the sensitive issue of how to treat the top moderated assessment. In high-stakes systems the results of these able students come under scrutiny and the resulting moderated assessment must be seen to be fair. An important category involves those students who were ranked first on the school assessment and also gained the top examination mark in their school groups. For such a student it would seem a reasonable principle that their moderated assessment should be no lower than the exam mark that they earned. If this principle is adopted, then this may require a further adjustment to raise their moderated assessment. Some systems may decide to automatically adjust the top moderated assessment to be equal to the top examination mark scored in the school group, regardless of who scored it (e.g. NSW Education Standards Authority, 2026).

Williamson (2016) notes the lack of transparency of several of the moderation methods in her survey. Although the mathematics presented earlier may seem complicated, schools need not be concerned with the details. The actual equation that performs the moderation (Equation 3) is a simple formula. The deriving of the constants a, b and c would be done by the central authority's computer. Most teachers would be able to understand the general principle of the Least Squares method of deriving a moderation conversion line if it is presented clearly through a diagram, without needing to consider the estimation details.

At the school level, teachers could receive a list of the raw assessment, moderated assessment and exam mark for each student, with students whose moderated assessment and exam mark are significantly different being flagged. The authority could also provide graphs of the form presented earlier in this paper. Schools would receive the constants a, b and c for each school group so that teachers who desired to do so could use Equation (3) to confirm the results and enhance their understanding of how the moderation worked. This paper has developed a viable model for the moderation of school assessments whose principles are not difficult to grasp and can accommodate a curvilinear relationship between the raw assessments and criterion scores.

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