
RESEARCH ARTICLE

Flexural Wave Band-Gap Characteristics of Metamaterial Beams with Membrane-Type Resonators: A Finite Element Investigation

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ABSTRACT

Membrane-type locally resonant metamaterials (MAMs) offer a potential approach for controlling structural vibration through frequency-selective modification of elastic-wave propagation. This study numerically investigates the flexural-wave dispersion and dynamic response of a metamaterial beam incorporating membrane-type resonators. Three-dimensional finite element models were developed in COMSOL Multiphysics V5.2 and analyzed using complementary eigenfrequency and frequency-domain approaches. A representative periodic unit cell was employed to determine the dispersion characteristics, while a finite four-cell structure was used to examine the frequency-dependent structural response. Aluminum 3003-H18 and AISI 4340 steel were used as constituent materials. Periodic boundary conditions and a parametric wave-number sweep were applied in the unit-cell analysis. The calculated dispersion relation exhibited flexural-wave band-gap behavior within the investigated frequency range extending to approximately 28 kHz, with the first reported band-gap feature occurring at approximately 4 kHz. Representative eigenmodes showed substantial changes in deformation pattern with frequency, supporting the interpretation that the observed dispersion characteristics are associated with the dynamic interaction between the host structure and local resonators. Frequency-domain analysis of the finite four-cell beam revealed pronounced frequency-dependent variations in displacement amplitude and deformation pattern. A notable response feature was observed in the approximately 20–25 kHz range and was qualitatively consistent with the unit-cell dispersion characteristics. Overall, the numerical results demonstrate that periodically distributed membrane-type resonators can substantially modify the flexural-wave characteristics of a supporting beam. The study provides a numerical basis for further investigation of membrane-resonator structures for frequency-selective vibration control, including future studies incorporating mesh convergence, damping, experimentally characterized membrane properties, quantitative transmission metrics, and experimental validation.

KEYWORDS

Acoustic metamaterial; membrane-type resonator; phononic crystal; flexural wave; band gap; finite element method; eigenfrequency; frequency response; COMSOL Multiphysics; vibration control

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1. Introduction

Vibration and low-frequency noise remain persistent challenges in mechanical and transportation structures, including buildings, machinery, automobiles, and aircraft. Dynamic excitation from engines, rotating machinery, aerodynamic loading, and other sources can generate structural vibration and airborne sound that adversely affect passenger comfort, acoustic performance, structural reliability, and the operation of sensitive equipment. In lightweight engineering structures, however, conventional approaches to vibration and noise reduction are constrained by the competing requirements of acoustic performance, structural mass, and available installation space. Traditional passive noise-control methods generally rely on increasing structural mass, increasing panel thickness, introducing porous or viscoelastic materials, or employing multilayer configurations. According to the conventional mass-law relationship, substantial increases in surface density are required to obtain appreciable improvements in sound transmission loss. It notes that approximately doubling the surface mass of a mass-controlled panel yields only about a 6 dB increase in transmission loss. Double-wall configurations can provide improved insulation, but they come with additional

mass, structural thickness, and resonance-related limitations (Qian et al., 2013; Zhou & Crocker, 2010). These disadvantages are particularly important in aerospace and automotive applications, where weight and space are tightly constrained.

Periodic structures provide an alternative approach in which wave propagation is controlled by the structure's dynamic characteristics. Phononic crystals are artificially structured materials containing periodically distributed inclusions or scatterers that can exhibit frequency intervals in which elastic or acoustic-wave propagation is prohibited or strongly modified. These intervals are commonly referred to as band gaps. The underlying principles and applications of phononic crystals for manipulating elastic and acoustic waves have been extensively documented (Olsson & El-Kady, 2009; Lu et al., 2009; Pennec et al., 2009), while related periodic acoustic structures have also been developed for sound-control applications (García-Chocano et al., 2012).

Conventional phononic-crystal band gaps are frequently associated with Bragg scattering, for which the characteristic periodic dimensions are related to the wavelength of the propagating wave. Consequently, achieving attenuation at relatively low frequencies may require comparatively large periodic structures. The development of locally resonant metamaterials provided an important alternative because locally resonant inclusions can interact with wavelengths substantially larger than the characteristic dimensions of the unit cell. Locally resonant sonic materials were shown to exhibit unusual effective dynamic properties and low-frequency stop-band behavior arising from resonant internal structures (Liu et al., 2000; Li & Chan, 2004; Lee et al., 2009). This concept has subsequently become an important basis for the design of acoustic and elastic metamaterials (Ma & Sheng, 2016). Among locally resonant systems, MAMs are particularly attractive for lightweight vibration and noise-control applications. A typical membrane-type resonator consists of a thin membrane supported by a surrounding frame and coupled to one or more concentrated masses. The membrane and attached mass form a resonant subsystem whose dynamic response can differ substantially from that predicted solely by the structure's static mass. Experimental and theoretical studies have demonstrated that membrane-type metamaterials can produce strong frequency-selective acoustic responses despite having relatively small surface mass (Yang et al., 2008; Naify et al., 2010; Yang et al., 2010).

The dynamic behavior of membrane-type resonators is governed by the interaction among membrane deformation, concentrated-mass motion, structural constraints, and resonance/anti-resonance phenomena. In particular, different resonant states can involve either the coupled motion of the membrane and attached mass or pronounced membrane deformation with comparatively limited motion of the concentrated mass. Between resonant states, modal contributions may partially cancel in terms of spatially averaged displacement, producing an anti-resonant response and potentially reducing acoustic radiation or structural transmission (Yang et al., 2008; Naify et al., 2010). Such behavior provides a mechanism for achieving frequency-selective attenuation without relying exclusively on large structural mass. The lightweight nature of these resonant structures makes them potentially attractive for transportation applications, where lightweight acoustic metamaterial concepts have also been investigated for sound isolation (Sui et al., 2015). Aircraft and automotive structures, for example, are exposed to tonal and broadband excitation from engines, propellers, rotating components, aerodynamic loading, and structural vibration. Conventional acoustic treatments can impose undesirable penalties in terms of mass and occupied volume. Locally resonant elastic metamaterials can control guided and flexural waves through subwavelength resonant mechanisms (Chen et al., 2011; Pai et al., 2014; Zhu et al., 2014; Sugino et al., 2016; Chen et al., 2017). Periodically distributed resonators can modify the dispersion characteristics of beam- and plate-like structures and generate frequency-selective band gaps associated with local resonance (Oudich et al., 2011; Xiao et al., 2012; Pai et al., 2014; Sugino et al., 2016). For membrane-type systems, membrane geometry, attached mass, resonator configuration, and membrane properties strongly influence frequency-dependent attenuation characteristics (Langfeldt et al., 2015, 2016; Huang et al., 2016; Chen et al., 2017).

A periodically distributed resonator system could therefore provide an alternative means of modifying structural-wave propagation while maintaining relatively low added mass. Nevertheless, the practical effectiveness of such a system depends strongly on resonator geometry, material properties, resonant mass, periodic spacing, boundary conditions, and the resulting coupling between local resonances and the supporting structure (Langfeldt et al., 2016; Chen et al., 2017).

For periodic metamaterial beams, the relationship between frequency and wave number provides an important means of identifying frequency regions associated with altered wave propagation. A representative periodic unit cell can be analyzed using Bloch-type periodic conditions to obtain the corresponding dispersion relation. Frequency ranges without appropriate propagating branches may then be interpreted as band-gap regions. In addition to unit-cell eigenfrequency analysis, the forced response of a finite multi-cell structure can provide complementary information regarding frequency-dependent deformation and displacement response. The combination of these two approaches, therefore, provides a useful numerical framework for

investigating the dynamic behaviour of periodically resonant beams (Sugino et al., 2016; Chen et al., 2017). Finite element analysis (FEA) is particularly suitable for such investigations because membrane-resonator structures can involve complex geometries, multiple materials, periodic boundary conditions, concentrated resonant components, and strongly frequency-dependent deformation patterns. Numerical eigenfrequency analysis can be used to construct dispersion relations and examine representative mode shapes, whereas frequency-domain analysis can be employed to investigate the response of a finite structure under harmonic excitation.

Accordingly, the objective of the present study is to numerically investigate the flexural-wave band-gap characteristics of a metamaterial beam incorporating membrane-type resonators. A three-dimensional finite element model is developed in COMSOL Multiphysics V5.2. A representative periodic unit cell is first analyzed through an eigenfrequency formulation and wave-number parametric sweep to obtain the dispersion characteristics and identify the reported band-gap features. Representative eigenmodes are subsequently examined to interpret the associated deformation behavior. Finally, a finite four-cell metamaterial beam is analyzed in the frequency domain to evaluate its frequency-dependent structural response and qualitatively compare it with the unit-cell dispersion characteristics.

2. Theoretical Background

2.1 Flexural vibration of beams

For a slender beam undergoing small transverse deformation, the classical Euler-Bernoulli beam model provides a useful description of flexural vibration. Under the assumptions of linear elastic behavior, small deformation, and negligible transverse shear deformation, the static relation between distributed transverse load and beam deflection can be represented by $(EI \frac{d^4 w}{dx^4}) = q(x)$, where E is Young's modulus, I is the second moment of area of the beam cross-section, w denotes transverse displacement, and $q(x)$ is the distributed load.

For free transverse vibration of a uniform beam, inertia must also be considered. The governing relation can be expressed as $(\partial^4 w / \partial x^4) + \rho A (\partial^2 w / \partial t^2) = 0$, where ρ is material density, and A is the cross-sectional area. A separated solution yields spatial eigenfunctions and natural frequencies that are determined by the boundary conditions. A metamaterial beam containing membrane resonators is more complex than a uniform beam because additional local degrees of freedom alter the effective dynamic stiffness and inertia, but classical beam vibration remains the physical basis for interpreting the propagating flexural waves that interact with the resonators. Related beam-based metamaterial analyses similarly use flexural-wave formulations to interpret resonator-induced band-gap behavior (Pai et al., 2014; Chen et al., 2017).

2.2 Periodicity and band-gap formation

When identical resonant cells are repeated periodically along a beam, wave propagation may be analyzed using a representative cell with suitable periodic conditions. The displacement fields at the corresponding boundaries of adjacent cells differ by a phase shift governed by the Bloch wavenumber. In one-dimensional form, $u(x+a) = u(x) \exp(ika)$, where a is the lattice constant and k is the wave number. This Bloch-periodic approach is widely used in phononic-crystal and locally resonant band-structure analyses (Lu et al., 2009; Pennec et al., 2009; Oudich et al., 2011).

For each selected value of k , the solution of the structural eigenvalue problem produces a number of eigenfrequencies. Plotting these frequencies against wavenumber yields the dispersion relation. Continuous branches correspond to admissible wave modes, while frequency ranges lacking a propagating solution constitute band gaps. Such dispersion-based identification of stop bands is central to the analysis of locally resonant elastic metamaterials (Sugino et al., 2016; Chen et al., 2017).

Two broad mechanisms may contribute to band-gap formation. The first is Bragg scattering, which is primarily associated with destructive interference arising from periodicity. The second is local resonance, in which resonating substructures undergo strong motion relative to the host structure. In locally resonant metamaterials, attenuation can therefore occur at frequencies for which the wavelengths are significantly larger than the lattice constant. The membrane-plus-mass subsystem represents such a local resonator. Near resonance, large relative displacements can occur within the unit cell, causing the host structure to experience an effective dynamic inertia that differs distinctly from its static mass. Between neighbouring resonances, anti-resonant motion can reduce the averaged motion or transmission. This dynamic interaction provides the principal physical basis for the attenuation behavior investigated in the present model. The distinction between Bragg-type and local-resonance mechanisms, including effective dynamic-mass effects, is well established in acoustic and elastic metamaterials (Liu et al., 2000; Lu et al., 2009; Ma & Sheng, 2016; Sugino et al., 2016).

2.3 Membrane-type acoustic metamaterials

A membrane-type resonator generally consists of a thin membrane supported along its boundaries and carrying an additional concentrated mass. Its response is governed by the membrane stiffness or tension, the membrane mass, the attached mass, the geometry, and the boundary constraints. Although the current finite element model employs solid-mechanics representations of the constituent parts, the underlying concept is analogous to the conventional MAM configuration. Comparable membrane-mass configurations have been reported in both experimental and analytical studies of membrane-type acoustic metamaterials (Yang et al., 2008; Naify et al., 2010; Huang et al., 2016).

Two characteristic resonant states are particularly important. In one mode, the membrane and attached mass move largely together. In another, the membrane undergoes pronounced motion while the attached mass participates much less strongly. The response between these two states may produce an anti-resonance in which the modal contributions to the spatially averaged displacement nearly cancel. Because acoustic radiation and structural transmission are strongly related to surface motion, the anti-resonance can generate unusually high attenuation. The mechanism, therefore, differs from simple mass loading: the resonator redistributes vibration among coupled degrees of freedom, creating a frequency-dependent effective response. Resonance, anti-resonance, mass loading, and membrane-property effects have been documented as key determinants of membrane-type metamaterial transmission behavior (Yang et al., 2008; Naify et al., 2010; Langfeldt et al., 2015, 2016).

3. Methodology

3.1 Numerical platform

The numerical models were constructed in COMSOL Multiphysics V5.2 using the three-dimensional Solid Mechanics interface. Two complementary analyses were performed. First, an eigenfrequency model of a representative periodic cell was used to obtain the dispersion characteristics. Second, a finite multi-cell model was analyzed in the frequency domain to evaluate the response of the structure over the investigated frequency range. Eigenfrequency analysis provides the natural modes associated with individual values of the wave-number parameter and is therefore appropriate for band-structure calculation, whereas frequency-domain analysis represents the forced harmonic behavior of a finite system and allows the resulting displacement distribution to be observed directly.

3.2 Unit-cell geometry and geometric parameters

The fundamental geometric parameter in the unit-cell calculation was the lattice dimension $a = 0.08$ m. The principal model parameters reported in the research were $h_0 = 0.01$ m, $b = 0.01$ m, $a_1 = a = 0.08$ m, and $h_b = 2b = 0.02$ m. The parameter n , used in discretizing the wave-number sweep, was set to 10, while the initial value of k_x was zero.

The three-dimensional unit-cell geometry consisted of five domains and contained 36 boundaries, 68 edges, and 40 vertices. The structure was assembled from block primitives representing the host, resonator, and associated structural regions. The first principal block had plan dimensions a by a_1 and height $h_0/2$. Another structural component had dimensions approximately $(a-0.015)$ by $(a_1-0.015)$ and height $4h_0$. The central resonant block had in-plane dimensions b by b and height h_b , while a thin component used dimensions $(a-0.015)$ by $(a_1-0.015)$ and a thickness corresponding to $h_0/30$. These values define a compact periodic cell containing a relatively small local feature compared with the overall lattice dimension. The numerical models employed in this study are illustrated in Figure 1. A representative unit cell was used for the eigenfrequency and dispersion analysis (Figure 1a), whereas a finite four-cell metamaterial beam was constructed for the frequency-response analysis (Figure 1b). The corresponding finite-element discretization is shown in Figure 1(c).

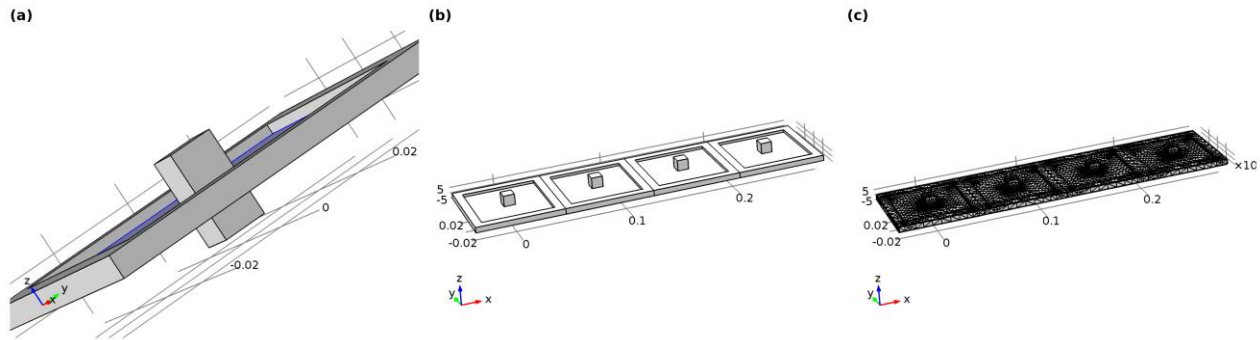


Figure 1. Numerical models used in the present study: (a) representative unit-cell model used for eigenfrequency and dispersion analysis, (b) finite four-cell metamaterial beam used for frequency-response analysis, and (c) finite-element mesh of the four-cell model

3.3 Material properties

Two solid materials were used in the numerical model: Aluminum 3003-H18 and AISI 4340 steel. For Aluminum 3003-H18, the material properties employed in the finite multi-cell model were density $\rho = 2730 \text{ kg/m}^3$, Young's modulus $E = 69 \text{ GPa}$, and Poisson's ratio $\nu = 0.33$. For AISI 4340 steel, the parameters were $\rho = 7850 \text{ kg/m}^3$, $E = 205 \text{ GPa}$, and $\nu = 0.28$. The material properties assigned to the constituent domains are summarized in Table 1.

Table 1: Material properties

Material	Density (kg/m^3)	Young's modulus (GPa)	Poisson's ratio
Aluminum 3003-H18	2730	69	0.33
AISI 4340 steel	7850	205	0.28

The contrast in material density and stiffness allows the resonant components and supporting regions to interact dynamically over the frequency range of interest. In the unit-cell implementation, aluminium and steel were assigned to distinct geometric domains based on their structural roles.

3.4 Periodic conditions and wave-number sweep

Periodic conditions were applied to the representative unit-cell model to evaluate the wave-propagation characteristics of the metamaterial beam. The periodic-condition configuration used in the finite element model is illustrated in Figure 2. The displacement field was represented using quadratic interpolation, and complex-valued variables were employed in the periodic-wave analysis. The dispersion relation was calculated by applying periodic conditions to the unit-cell model. The displacement interpolation was quadratic, and complex-valued variables were enabled as required for periodic-wave calculations. The wavenumber parameter k_x was varied over the range $k_x = \text{range}(0, \pi/(an), \pi/a)$, with $n = 10$. Thus, the model sampled the wave-number interval from the center to the edge of the one-dimensional irreducible Brillouin zone. For every wave-number value, the eigenfrequency solver was used to calculate the natural frequencies of the cell. The resulting eigenfrequencies were then plotted against k_x to form the dispersion diagram. This method permits direct identification of frequency regions where no relevant propagating flexural branch is present.

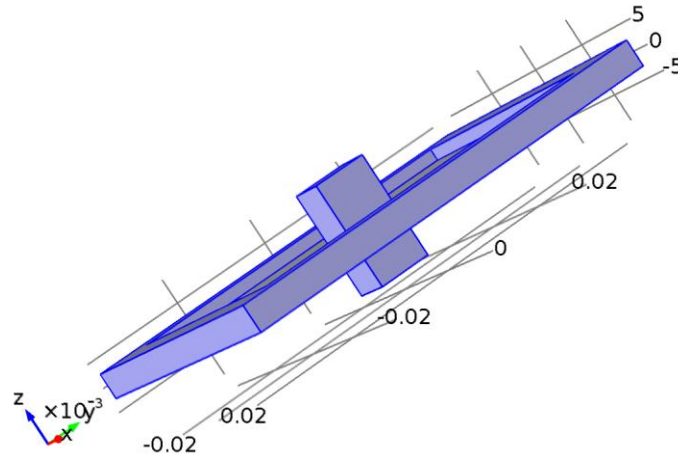


Figure 2. Periodic-condition configuration of the representative unit-cell model used for eigenfrequency and dispersion analysis

3.5 Structural constraints

The unit-cell solid-mechanics model contained periodic conditions and prescribed-displacement constraints. The prescribed displacement settings constrained the x- and y-direction displacement components while allowing the transverse deformation required to capture flexural behavior. For the finite beam, a larger structure containing repeated cells was created. Four repetitions were generated using an array with a displacement of a along the beam direction. The resulting geometry contained 20 domains, 141 boundaries, 260 edges, and 148 vertices. This model represented a finite section of the periodic metamaterial rather than an isolated unit cell.

3.6 Meshing

The eigenfrequency unit-cell model employed a three-dimensional finite-element mesh comprising 2330 tetrahedral elements, 1496 triangular surface elements, 292 edge elements, and 40 vertex elements. The reported average element quality was 0.1414, while the minimum element quality was 3.28×10^{-5} . The prescribed maximum and minimum element sizes were approximately 0.04 m and 0.0056 m, respectively. The larger frequency-response model was more heavily discretized. It contained 24,415 tetrahedral elements, 11,884 triangular elements, 1,890 edge elements, and 148 vertex elements. Its average element quality was 0.3893 and its reported minimum element quality was 4.18×10^{-5} . The corresponding maximum element size was 0.032 m, the minimum element size was 0.00576 m, the curvature factor was 0.6, the resolution of narrow regions was 0.5, and the maximum element growth rate was 1.5. Accordingly, the numerical trends reported here should be interpreted as results from the specified finite element discretization rather than as mesh-independent benchmark values.

3.7 Eigenfrequency and frequency-response analyses

The eigenfrequency analysis was performed first on the periodic unit cell. At every wave-number value, a set of natural frequencies and associated deformation patterns was calculated. The mode-shape visualization reported one example with a complex eigenfrequency of approximately $10779 + 2139.8i$ in the COMSOL result display. Multiple eigenmodes were examined at different frequencies to identify changes in the deformation pattern. The eigenfrequency results were then assembled into a one-dimensional frequency-versus-wave-number plot, which served as the primary basis for identifying regions of attenuation. The second analysis employed a four-cell finite-metamaterial beam. Aluminium 3003-H18 was assigned to domains 2, 4, 7, 9, 12, 14, 17, and 19, while AISI 4340 steel was assigned to the remaining listed structural domains. Frequency-domain solutions were obtained across the range considered in the study. The computed quantities included deformation patterns, von Mises stress distributions, and the z-component of displacement amplitude at a selected observation point.

The principal numerical parameters adopted for the eigenfrequency and frequency-response analyses are summarized in Table 2. The representative unit-cell model was employed to determine the eigenfrequencies and dispersion characteristics of the periodic metamaterial structure, whereas the finite four-cell model was used to investigate its frequency-dependent structural response. The two models were discretized independently according to their respective geometric and computational requirements.

Table 2. Numerical and finite element analysis settings used in the present study

Parameter	Unit-cell / eigenfrequency model	Finite four-cell / frequency-response model
Software	COMSOL Multiphysics V5.2	COMSOL Multiphysics V5.2
Physics interface	Solid Mechanics, 3D	Solid Mechanics, 3D
Analysis type	Eigenfrequency analysis	Frequency-domain analysis
Number of cells	1 representative periodic unit cell	4 repeated cells
Lattice constant, (a)	0.08 m	0.08 m
Displacement interpolation	Quadratic	Quadratic
Periodic condition	Applied	Applied/continuity-type condition
Complex variables	Enabled	—
Prescribed displacement	($u_x=0$;; $u_y=0$); (u_z) unconstrained	Prescribed displacement applied to selected domains
Wave-number parameter	(k_x)	—
Wave-number range	($0 \leq k_x \leq \pi/a$)	—
Wave-number discretization parameter, (n)	10	—
Mesh type	Tetrahedral	Tetrahedral
Tetrahedral elements	2,330	24,415
Triangular elements	1,496	11,884
Edge elements	292	1,890
Vertex elements	40	148
Maximum element size	0.040 m	0.032 m
Minimum element size	0.0056 m	0.00576 m
Average element quality	0.1414	0.3893
Minimum element quality	(3.28×10^{-5})	(4.18×10^{-5})
Maximum element growth rate	—	1.5
Curvature factor	—	0.6
Resolution of narrow regions	—	0.5
Primary output	Eigenfrequencies, mode shapes and dispersion relation	Displacement response, deformation/mode shapes and stress response

4. Results

4.1 Eigenfrequency dispersion characteristics

The calculated dispersion diagram demonstrated that the membrane-type resonators significantly altered the flexural-wave characteristics of the host beam. The investigated eigenfrequency range extended from approximately 1 Hz to 28 kHz. According to the reported band-structure interpretation, the first flexural band-gap feature appeared at approximately 4 kHz, while additional attenuation behavior occurred within the frequency range extending toward 28 kHz.

The importance of this result lies not simply in the numerical value of the lower reported gap boundary, but in the existence of stop-band behavior produced by the resonant architecture. A homogeneous beam typically supports continuous families of flexural modes, with the relationship between wavenumber and frequency determined primarily by its flexural rigidity and distributed mass. Introducing periodically arranged membrane-resonator units produces additional local degrees of freedom. These degrees of freedom interact with the host-beam motion, causing branches in the dispersion relation to bend, flatten, split, or terminate around resonant frequencies.

Previous research notes that relatively flat branches were disregarded when identifying the relevant flexural band gap. This is an important distinction because a periodic resonant system may display locally confined modes that exhibit little change in frequency as a function of wavenumber. A nearly horizontal branch can represent a resonant mode dominated by motion of the

local attachment rather than efficient energy transport along the beam. Consequently, interpretation of a band structure should be based not only on visual gaps between all eigenfrequency branches but also on the propagation character of the individual modes. The calculated dispersion relation of the membrane-type metamaterial beam is presented in Figure 3. The numerical results indicate the formation of flexural-wave stop-band regions within the investigated frequency range, with the first reported band-gap feature appearing at approximately 4 kHz. Similar distinctions between propagating branches and locally resonant/weakly dispersive modes have been used in interpreting locally resonant band structures (Sugino et al., 2016; Chen et al., 2017).

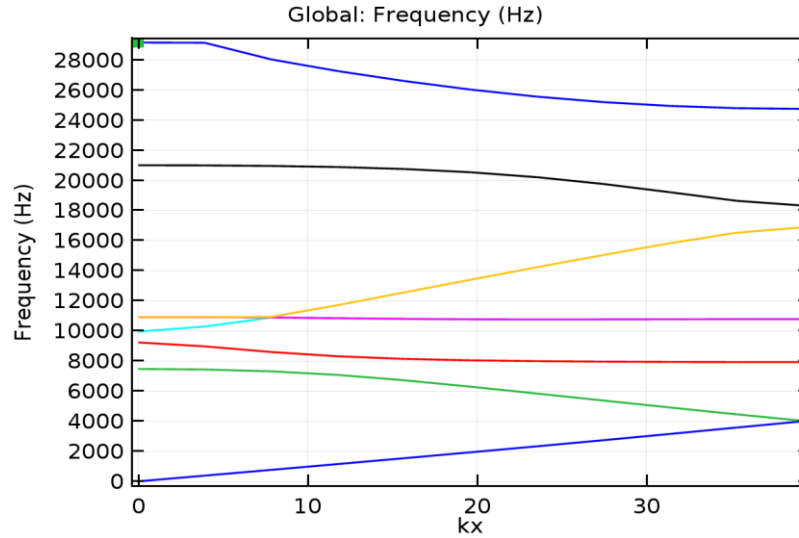


Figure 3. Calculated dispersion relation of the membrane-type resonator beam obtained from the eigenfrequency analysis. The dispersion branches indicate the formation of flexural-wave band-gap regions within the investigated frequency range.

4.2 Eigenmode behavior

Representative deformation patterns reveal the mechanism underlying the dispersion characteristics. The calculated modes vary substantially with eigenfrequency. At certain frequencies, deformation is concentrated in the membrane or resonant part of the unit cell, whereas at other frequencies the supporting beam participates more strongly.

This distinction is central to locally resonant behavior. When the resonator moves strongly relative to the host structure, energy becomes concentrated within the cell rather than transmitted efficiently from one cell to the next. The local dynamic inertia generated by this motion can oppose the supporting beam's motion, thereby reducing the net displacement and inhibiting propagation. The mode-shape results, therefore, complement the dispersion diagram by showing a corresponding transition in structural motion around the attenuation regions. To clarify the physical mechanism associated with the predicted band gaps, representative eigenmodes near the relevant dispersion branches were examined. Figure 4 compares the corresponding deformation patterns and demonstrates substantial changes in the relative motion of the host beam. The mode shapes in Figure 2 indicate that some modes are dominated by global flexural deformation of the supporting structure, whereas others exhibit stronger localized deformation around the resonator. This transition supports the interpretation that local resonance contributes to the formation of the flexural-wave attenuation regions. This localization-based interpretation is consistent with established local-resonance mechanisms in finite elastic metamaterials and membrane-mass beams (Sugino et al., 2016; Chen et al., 2017).

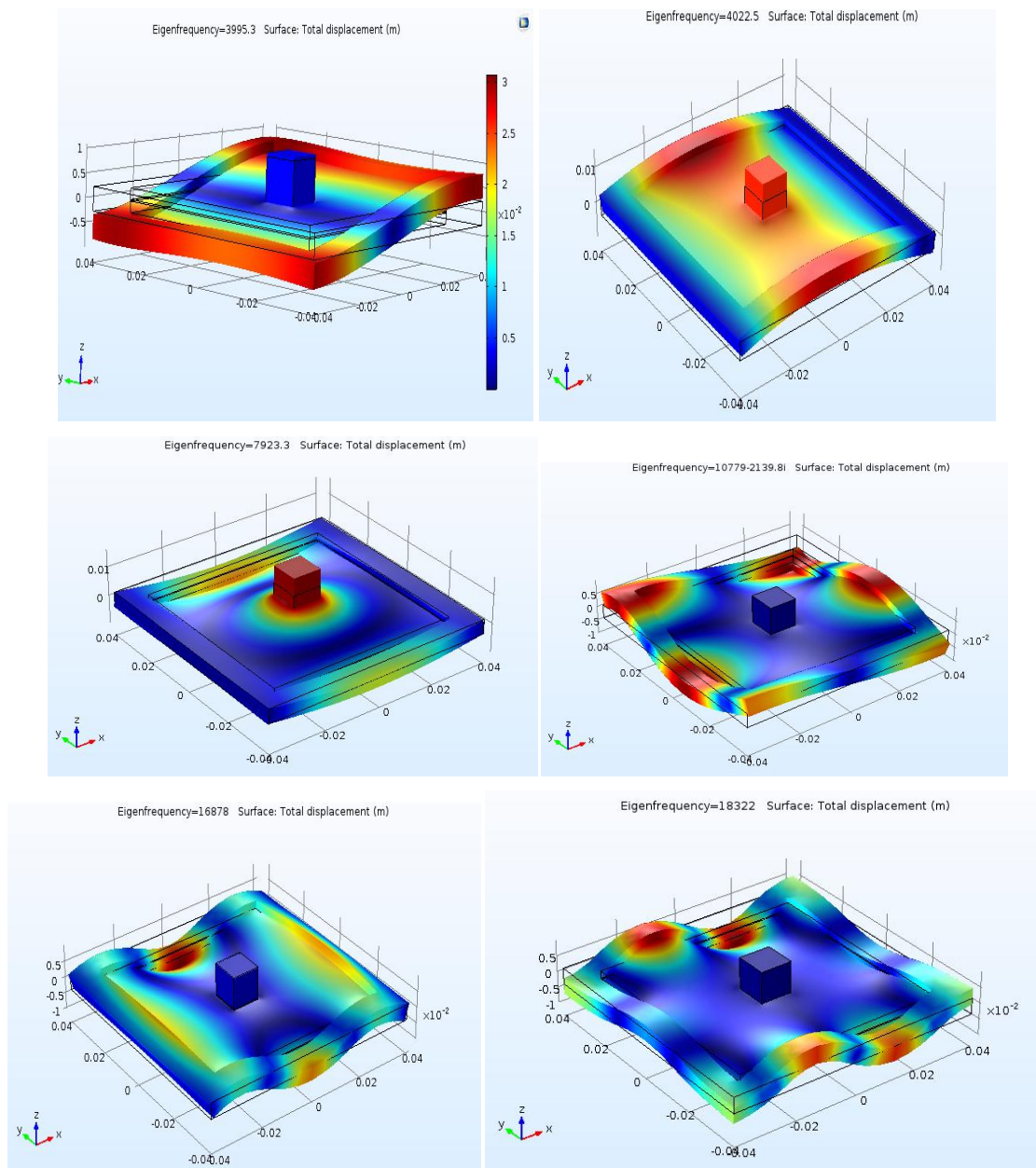


Figure 4: Mode shapes of the eigen-modes labelled

4.3 Frequency-response behavior of the finite metamaterial beam

A finite four-cell structure was subsequently evaluated in the frequency domain. The calculated mode shapes showed that the global deformation pattern changed significantly as the excitation frequency increased. At certain frequencies, relatively long-wavelength bending occurred across the entire beam. At others, deformation became more spatially complex and increasingly influenced by individual resonator cells. A representative stress result was shown at a frequency near 9951 Hz, where the beam exhibited a periodic flexural deformation pattern across the resonant cells. The calculated mode-shape sequence further demonstrated that changes in excitation frequency could shift both the magnitude and spatial distribution of deformation. The displacement-frequency response was particularly informative. The plotted z-component of the displacement amplitude exhibited multiple resonant peaks at lower and intermediate frequencies, followed by pronounced reductions in response over selected intervals.

This comparison is useful because the two calculations represent different physical problems. The unit-cell analysis assumes an infinitely periodic medium, based on periodic phase relations, and yields the theoretical propagating modes of the periodic system. The finite-beam frequency-response model contains only a small number of repeated cells and is therefore susceptible

to boundary effects, discrete resonances, and finite-size effects. Agreement between prominent attenuation or transition features in these two analyses provides qualitative support for the predicted band-gap behavior. The forced-response calculations also revealed pronounced changes in deformation pattern as the excitation frequency was varied. Representative responses are compared in Figure 5. As illustrated in Figure 5, the response evolves from relatively global bending deformation to increasingly localized and spatially complex patterns as the excitation frequency changes. The frequency dependence of the structural response was further evaluated from the displacement amplitude at the selected observation point. The corresponding response spectrum is presented in Figure 6 and shows a sequence of resonant peaks followed by pronounced reductions in displacement amplitude over selected frequency intervals. In particular, the substantial change in response in the approximately 20–25 kHz range is qualitatively consistent with the band-structure behavior obtained from the unit-cell eigenfrequency analysis. The distinction between infinite-periodic dispersion predictions and finite-structure response is also emphasized in finite locally resonant metamaterial studies (Sugino et al., 2016).

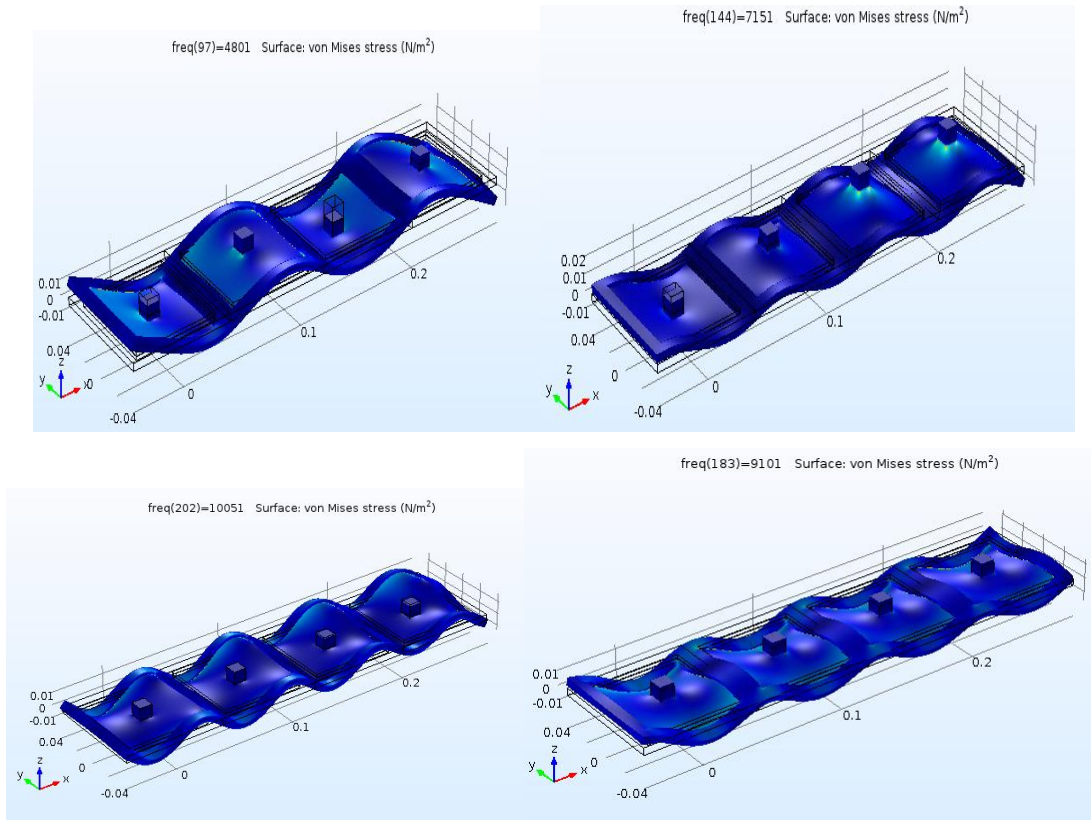


Figure 5: Calculated deformation patterns of the finite metamaterial beam at selected excitation frequencies: (a) f_1 , (b) f_2 , (c) f_3 , and (d) f_4

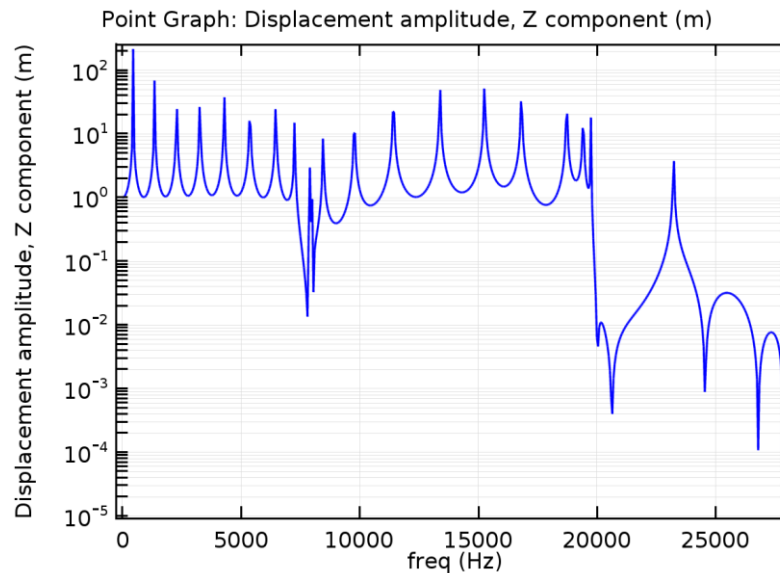


Figure 6. Frequency-dependent deformation patterns of the finite membrane-type metamaterial beam obtained from the frequency-domain analysis

4.4 Relationship between band-gap boundaries and resonant modes

Band-gap edge frequencies in periodic resonant systems frequently correspond to characteristic standing-wave configurations or local-resonance modes of the unit cell. As the wavenumber approaches a band edge, the group velocity may decrease, and the deformation pattern may become strongly localized. Once the excitation enters an attenuation region, propagation from one unit to the next becomes inefficient. Beyond the band gap, a new propagating branch may appear with a different mode shape. Band-edge localization and resonator-dominated standing-wave behavior have been reported in locally resonant beam and finite-metamaterial systems (Pai et al., 2014; Sugino et al., 2016; Chen et al., 2017).

The local membrane resonator can thus be viewed as a dynamic filter embedded within the structural load path. At frequencies far from resonance, the cell participates in host-beam vibration relatively normally. Near local resonance, however, the relative phase between resonator and host motion changes rapidly. This alters the effective dynamic mass and stiffness, creating frequency intervals in which flexural-wave propagation is strongly suppressed. The associated changes in effective dynamic mass and stiffness are fundamental features of locally resonant acoustic and elastic metamaterials (Liu et al., 2000; Yang et al., 2008; Ma & Sheng, 2016).

5. Discussion

5.1 Significance of the flexural band gap

The numerical results demonstrate that adding membrane-type resonators to a beam can change its dispersion behavior fundamentally. Instead of relying exclusively on damping or additional structural mass, the proposed architecture creates frequency-selective attenuation through local dynamic interaction. Such behavior is particularly relevant to noise and vibration sources dominated by discrete tonal components. Propeller-driven aircraft, rotating machinery, engines, and many electromechanical systems generate vibration spectra containing strong peaks at fundamental frequencies and harmonics. If the resonator system is designed so that a band gap overlaps one or more of these dominant frequencies, significant attenuation may be achievable without a proportionate increase in structural mass. Comparable frequency-selective vibration suppression has been demonstrated in resonator-based metamaterial beams and related elastic structures (Pai et al., 2014; Zhu et al., 2014; Chen et al., 2017).

The concept, therefore, differs from that of conventional broadband insulation. A mass-loaded barrier tends to improve attenuation as surface density increases, whereas a resonant metamaterial can produce a much stronger effect within a deliberately targeted frequency range. The disadvantage is that the attenuation bandwidth may be limited. Consequently, the

practical design problem becomes one of matching resonator properties to the relevant excitation spectrum. The observed transition between global structural deformation and localized resonator-dominated motion is consistent with the local-resonance mechanisms reported for other flexural-wave metamaterial systems. This interpretation is consistent with reported local-resonance mechanisms in finite elastic and membrane-mass metamaterial systems (Sugino et al., 2016; Chen et al., 2017).

5.2 Role of membrane resonance

The membrane is essential because it provides a compliant element whose dynamic behavior can be manipulated by geometry, material properties, prestress, and attached mass. The concentrated mass lowers and modifies the resonant frequencies, while the membrane establishes the restoring force. In an idealized mass-spring interpretation, resonant frequency scales approximately with the square root of effective stiffness divided by effective mass. Increasing the attached mass would therefore generally move a local resonance toward a lower frequency, whereas increasing membrane stiffness or tension would tend to increase the resonant frequency. The underlying principle suggests several routes for engineering the band gap. Attached mass, membrane dimensions, membrane thickness, pretension, lattice constant, and host-beam stiffness can all be varied to shift or broaden the attenuation range. Multiple resonators with different characteristic frequencies could potentially be introduced within a single cell to create multiple or overlapping gaps. Previous studies indicate that resonator mass, membrane properties, geometric configuration, and coupled resonances provide effective parameters for tuning the frequency and bandwidth of membrane-type metamaterial responses. The influence of attached mass, membrane properties, geometry, and prestress on resonance tuning has been demonstrated in membrane-type metamaterials (Langfeldt et al., 2015, 2016; Huang et al., 2016; Chen et al., 2017).

5.3 Implications for low-frequency noise reduction

The motivation for membrane-type metamaterials is strongest at low frequencies, where conventional lightweight sound barriers are least effective. The mass-law limitation means that a large increase in conventional panel mass may be required to obtain only a modest improvement in transmission loss. Increasing double-wall spacing can also improve low-frequency behavior, but available space may prevent this modification in compact engineering systems. Related studies of sandwich panels and metamaterial plates likewise show that achieving transmission-loss improvements in lightweight structures requires careful control of dynamic, rather than purely static, mass effects (Zhou & Crocker, 2010; Xiao et al., 2012; Qian et al., 2013).

Membrane-type structures address the problem differently. Their performance is governed by resonance and anti-resonance rather than static mass alone. Membrane-based acoustic structures can achieve strong sound-insulation behavior below approximately 500 Hz in previously reported applications and identify membrane metamaterials as potential replacements for heavier conventional treatments. Strong low-frequency attenuation has been reported for membrane-type and related lightweight acoustic metamaterial panels (Naify et al., 2010, 2011; Yang et al., 2010; Sui et al., 2015; Huang et al., 2016).

The specific numerical frequencies obtained in the present beam model are considerably higher than the target range, extending through the kilohertz region. This does not invalidate the metamaterial concept; rather, it indicates that the modeled geometry and constituent properties produce resonances at those frequencies. For a practical low-frequency application, the geometry and effective resonator mass would need to be redesigned to shift the relevant dispersion features downward. The present model therefore demonstrates the mechanism of band-gap formation rather than an optimized aircraft-cabin or automotive treatment in its current form.

5.4 Comparison between dispersion and frequency-response analyses

The use of two numerical approaches strengthens the interpretation. A dispersion relation predicts the modes permitted by an idealized infinite periodic structure. It is particularly valuable for determining the fundamental wave physics of the metamaterial. However, a real structure contains a finite number of cells and is excited at boundaries or discrete points. The actual measured response therefore depends on how these propagating and evanescent modes interact with the finite geometry. This distinction between ideal periodic dispersion and finite-structure dynamics is a recognized issue in locally resonant metamaterial analysis (Sugino et al., 2016).

The frequency-response calculation addresses this second problem. The multi-cell beam exhibits resonant peaks where the applied harmonic excitation couples strongly to structural modes and reduced response where transmission through the repeated cells is limited. The reported correspondence around 20-25 kHz indicates that the band-structure characteristics remain visible even in a finite arrangement. For future studies, this comparison should be made quantitatively using a defined input-output transmission metric. Finite-beam and membrane-mass studies similarly use forced-response behavior to complement dispersion-based interpretations (Chen et al., 2017).

5.5 Design considerations for engineering applications

A practical metamaterial treatment must satisfy several requirements simultaneously. First, the attenuation band must overlap the dominant frequencies of the vibration or acoustic disturbance. Second, the design should maintain low mass and limited thickness. Third, sufficient damping may be needed to prevent excessive resonator amplitudes near resonance. Fourth, the structure should remain mechanically robust under static loading, environmental variation, and manufacturing tolerances. These competing design requirements are also evident in studies of lightweight and membrane-type acoustic metamaterials (Sui et al., 2015; Huang et al., 2016; Ma & Sheng, 2016).

The membrane-resonator concept offers useful flexibility because several parameters can be varied independently. The membrane can be modified through changes in thickness, pretension, material, or dimensions. The attached mass can be varied in magnitude and location, while the host lattice spacing may also be changed. Such flexibility enables tailoring the metamaterial to individual applications. A distributed array of resonators could, for example, be tuned to dominant cabin-noise frequencies without requiring changes to the external aerodynamic shape of an aircraft. Adjustable membrane properties and resonator parameters have been demonstrated as practical tuning variables in membrane-type metamaterial designs (Langfeldt et al., 2016; Chen et al., 2017).

5.6 Limitations of the present study

Several limitations should be recognized before drawing quantitative design conclusions. First, the finite element mesh used in the unit-cell calculation was described as extremely coarse in the original model settings, and no formal mesh-convergence study was reported. Band-edge frequencies can be sensitive to discretization, particularly for thin components or localized deformation. A publication-quality follow-up study should repeat the calculation using progressively refined meshes and demonstrate that the principal band boundaries converge.

Second, material damping was not systematically characterized. Real membranes and structural joints dissipate energy, and damping can strongly alter resonant peaks and anti-resonant minima. An undamped or weakly damped eigenfrequency calculation is suitable for identifying band-structure mechanisms, but realistic transmission predictions require experimentally measured loss factors or equivalent viscoelastic parameters.

Third, membrane prestress is central to the physics of conventional membrane-type acoustic metamaterials, yet neither a measured prestress value nor a corresponding tension-sensitivity analysis is documented in the available model description. Because membrane tension directly affects resonant frequency, future models should explicitly define and vary prestress. The sensitivity of membrane-resonator frequencies to prestress and related membrane parameters has been specifically documented in analytical and beam-based studies (Langfeldt et al., 2016; Chen et al., 2017).

Fourth, the numerical analysis uses idealized geometry and material interfaces. Bonding layers, manufacturing tolerances, attachment stiffness, and eccentricity of the concentrated mass could all influence the actual dynamic response. Fifth, the experimental results agree with theoretical predictions, but it does not provide sufficient experimental procedure and quantitative data to support a reproducible validation section. A journal submission should therefore either present a complete experimental methodology and measured dataset or describe the work explicitly as a numerical investigation. Finally, the frequency-response model contains only four repeated cells, and a systematic parameter-optimisation study. The configuration should therefore be treated as a proof-of-concept model rather than a final engineered design.

6. Prospects for Optimization

The present results indicate several directions for improving membrane-resonator performance. The most direct approach is to tune the concentrated mass. Increasing the resonant mass should generally decrease the local resonant frequency, thereby shifting the attenuation region toward the low-frequency range of greatest practical interest. However, excessive mass would undermine the lightweight advantage of the concept, so the design task involves maximizing the dynamic effect per unit added mass. Mass-based tuning of locally resonant beam and membrane systems is consistent with prior analytical and numerical findings (Pai et al., 2014; Langfeldt et al., 2015; Chen et al., 2017).

Membrane stiffness provides a second tuning variable. A softer or less highly tensioned membrane would generally shift resonance downward, whereas increased tension would shift it upward. Spatially graded membrane properties could potentially generate overlapping local resonances and broaden the effective attenuation band. The periodic spacing also influences dispersion. While a purely local-resonant gap is not governed solely by Bragg scattering, the interaction between lattice periodicity and local resonances may affect bandwidth and modal coupling. Optimization should therefore consider both the

local resonator and the global periodic structure. Membrane tension/stiffness and periodic geometry are established tuning parameters for membrane-mass and locally resonant systems (Langfeldt et al., 2016; Chen et al., 2017).

Another promising strategy is the use of multiple resonators within each cell. If individual masses or membrane sections are designed with slightly different resonant frequencies, their attenuation regions may overlap, generating a wider stop band than a single-resonance design. The geometry could also be adapted to achieve tunability via prestress or pressure-induced geometric stiffening. Multi-resonator concepts have also been used to create multiple or broadened attenuation bands in metamaterial beams and resonator arrays (Naify et al., 2011; Pai et al., 2014).

Optimization should ultimately be formulated as a multi-objective problem involving band-gap center frequency, bandwidth, transmission reduction, total added mass, maximum deformation, peak stress, and manufacturing constraints. Finite element analysis would remain an effective tool for evaluating candidate designs. Finite-element and analytical optimization frameworks are therefore appropriate for balancing attenuation bandwidth, mass, deformation, and structural constraints (Xiao et al., 2012; Chen et al., 2017).

7. Conclusions

A numerical investigation of flexural-wave propagation in a metamaterial beam incorporating membrane-type resonators has been presented. The structure was modelled in COMSOL Multiphysics V5.2 using three-dimensional finite element analysis. A periodic unit cell was first analyzed through an eigenfrequency formulation combined with a wave-number parametric sweep. A finite four-cell beam was subsequently investigated in the frequency domain. The study shows that membrane-type resonators substantially modify the flexural-wave dispersion of a supporting beam. The additional local resonant degrees of freedom alter the relationship between frequency and wave number and generate frequency regions in which normal flexural-wave propagation is inhibited. Within the investigated range extending to approximately 28 kHz, the original numerical study reports the first band-gap feature at approximately 4 kHz. Mode shapes provide physical evidence of local-resonance effects. Distinct changes in deformation patterns occur as the eigenfrequency varies, demonstrating a transition between host-beam-dominated motion and localized resonator motion. The finite-beam frequency-response results qualitatively support the dispersion analysis, including a prominent response transition in the approximately 20-25 kHz region, which is reported to be consistent with the calculated band structure. The attenuation mechanism is associated with resonance and anti-resonance rather than static mass alone. The membrane and attached mass interact dynamically with the host structure, thereby enabling significant modification of wave propagation without requiring the large structural mass typically associated with conventional low-frequency barriers. With suitable tuning, membrane-resonator arrays may therefore provide a passive strategy for reducing tonal vibration in vehicles, aircraft structures, machinery, and other systems subject to strict mass and thickness constraints. The present model should be regarded as a numerical proof of concept. Before practical application, further work is required to establish mesh-independent predictions, introduce realistic damping and membrane prestress, quantify transmission loss, determine sensitivity to geometry and material parameters, and validate the numerical predictions experimentally. Particular attention should be given to shifting the resonant behavior toward the low-frequency range most relevant to transportation noise while retaining the lightweight character of the metamaterial. Overall, the results demonstrate the potential of membrane-type resonators as structural building blocks for frequency-selective control of flexural waves.

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