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| RESEARCH ARTICLE

Sudoku Puzzle Difficulty Rating based on Fuzzy Logic

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| ABSTRACT

In this document, we propose a new fuzzy logic-based rating technique for Sudoku difficulty, which takes into account Sudoku instance parameters such as the number of filled cells as well as parameters relating to the distribution of filled numbers on the cells. This new technique is validated using historical data from a certificate paper [Mantere, 2008], which includes 45 Sudoku instances of all rank levels, three of each level, and the average/max time consumed in 100 runs using different algorithms for each instance. First and foremost, these instances were analyzed and parameterized, and their parameters were quantitatively analyzed to be considered in fuzzy logic. The instance parameters' correlation with their solving time is studied, and dimensionality reduction was performed on these as variables to ensure that no unnecessary variable was included in the study. As solving time parameters, the number of filled cells in the instance, the minimum number of filled cells in rows and columns, and the number of empty sub-squares (3*3) in the instance are all accepted. Because there should be a functional relationship between the Sudoku rank and the time required to solve it, a linear regression model was performed on the historical data between the old rank and the solving time, and the same regression model was performed on the new rank to validate it. As a result, a new clear and simple ranking technique that outputs more correlated ranks with the time required to solve Sudoku puzzles is validated.

| KEYWORDS

Fuzzy logic, Sudoku, correlation, regression.

| ARTICLE INFORMATION

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1. Introduction

Sudoku puzzle is a well-known Japanese game that is solved by determining the numbers that must fill the empty cells while satisfying a set of constraints; its goal is to fill a 9×9 grid with numbers so that each row and column, and 3×3 sub squares contain all of the digits between 1 and 9. Playing Sudoku on a daily basis helps to improve concentration and overall brain power.

Anyone who wants to play such a game will prefer to play at an instance level appropriate for his puzzle knowledge and experience.

Beginners, for example, would leave the game if they were surprised by a difficult level instance, whereas experts would prefer to see real challenges if they chose the difficult level.

Most Sudoku applications use ranks that are not based on scientific principles, and there have been few attempts to rank the difficulty of Sudoku. The majority of them concern the number of filled cells in the instance, with the most popular one considering the techniques required to solve the instance, which will be discussed in the following section of related work. After that, the historical data used to extract the input parameters of our new rating technique will be discussed, and the parameter extraction will be validated with quantitated techniques; after that, the fuzzy rating concept and parameters will be illustrated, and its output new rank will be compared with the old technique from the historical data.

2. Literature review

Almost all Sudoku puzzle research has focused on developing efficient algorithms for solving them. They then competed to improve their methodologies in various ways. Because Sudoku is an NP-Complete problem, its time complexity is exponential and needs a non-deterministic algorithm to solve it, and the researchers are challenged to find a polynomial time solution for one of these NP-Complete problems. Many algorithms have been developed to solve Sudoku puzzles. Graph Referencing Algorithm (GRA) optimization, Genetic Algorithm (GA), Simulated Annealing (SA), Harmony Search (HS), and Brute Force algorithm are some examples. [Chatterjee n.d] Pel'anek presented an overview and evaluation of the prediction of the Sudoku puzzle difficulty rating on a large human problem-solving data set in 2014; the computational model of human solving was the best model in his evaluation. The author considered two main factors in the difficulty rating of this article. The first factor is the complexity of the individual steps and the structure of dependency among steps. The authors described the metrics as being based on analysis solutions with relaxed constraints, which is a novel approach inspired by the phase transition phenomenon. There are two basic techniques in human solving that directly correspond to the puzzle rules: The naked single technique (also known as the singleton, single value, forced value, and exclusion principle): Because all other values occur in the cell's row, column, or sub-grid, only one value can be entered into the sewa'2w cell for a given cell (any other number would lead to a direct violation of rules). And the hidden single technique (also known as naked value or the inclusion principle): There is only one cell per unit (row, column, or sub-grid) that can contain a given value (all other placements would lead to a direct violation of rules). Simple Sudoku refers to Sudoku problems that can be solved by iterating these two techniques. Simple Sudokus are the most widely used puzzles and are rated as easy or mild. Most ranking techniques are based on randomized local improvement and parallel search using a population of candidate solutions - these approaches bear little resemblance to human problem solving and thus do not appear to be useful for predicting human behavior. [Kˇriv´anek, 2011]. They employ a randomized approach analogous to the main model in that, rather than computing the smallest number of steps required to refute a given value, they simply employ a randomized sequence of simple steps and count the number of steps required to reach an inconsistency. The variable (cell) with the lowest score is considered the easiest to fill, and the refutation score is used to rate the difficulty of an unknown logic technique. There was always at least one cell with a finite score in all of our considered Sudoku puzzles; for more complex problems, it may be necessary to further specify the model for the case where all refutation scores have value. When they tested this model, they discovered that the best results were obtained for puzzles of intermediate difficulty. For simple puzzles, there are numerous ways to fill cells, making it difficult to predict the exact order (in these cases, the order also differs among individual solvers). Difficult puzzles cannot be solved using the model's basic techniques, so the prediction is slightly worse. Figure 6 [Kˇriv´anek, 2011]

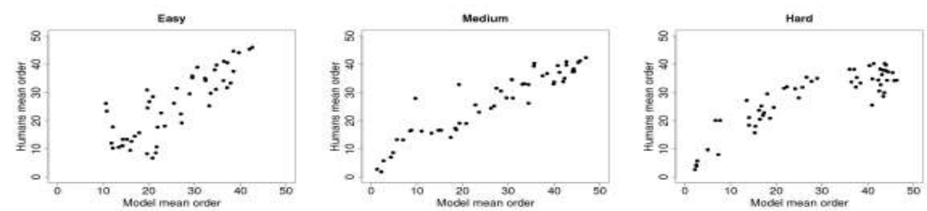


Fig. 6. Comparison of cell filling ordering by humans and by model for three sample puzzles of different difficulty. Each dot corresponds to one cell, the positions denote mean order of filling. Correlation coefficients: 0.84 (easy), 0.94 (medium), 0.86 (hard).

Some of this additional difficulty can be explained by the concept of 'dependency' among steps in the solution process (applications of logic techniques). An important aspect of human CSP solving is "the number of possibilities leading to a next step" in each step. The difficulty of logic techniques is determined by the work's Sudoku Explainer tool [Juillerat, 2007]. Only the eight most fundamental techniques are demonstrated. More than 20 techniques are classified by the tool. Because of their relational complexity, some of the simple techniques can be further characterized [Kˇriv´anek, 2011]. The tool developer provides this rating, which is usually based on personal experience and common knowledge. A sample of such ratings is shown in the table below.

Technique	Rating	Technique	Rating
Hidden single	1.2	Naked Single	2.3
Direct Pointing	1.7	Direct Hidden Triple	2.5
Direct Claiming	1.9	Pointing	2.6
Direct Hidden Pair	2.0	Claiming	2.8

Figure 1: Rating Technique

This approach has the disadvantage of containing a large number of ad hoc parameters and being highly Sudoku-specific, i.e., it provides limited insight into human problem-solving and is not transferable to other problems (the success of the approach is based on significant experience with the problem). There is another method for categorizing logic techniques. The method is based on the assumption that many advanced logic techniques are shortcuts for a search (what-if reasoning). As a result, they provide a difficulty rating for logic techniques using search. This method is not unique to Sudoku and has almost no parameters.

• They compute a "refutation score" for each unassigned variable (empty cell), which expresses the difficulty of assigning the correct value to this variable in the given state by rejecting all other possible candidates.

• For each incorrect candidate value v, they denote ref v as the smallest number of simple steps required to demonstrate the assignment's inconsistency.

• The "ideal refutation score" is calculated as the sum of ref v values.

• If some of the values cannot be refuted using simple steps, we set the score to.

The computation of ref v can be done by performing a breadth-first search over all possible puzzle states, but this is computationally expensive and does not correspond to human behavior. In general, the most popular Difficulty rating techniques rely on the number of full cells in the Soduko instance because it goes without saying that when the number of filled cells is larger, the Soduko is easier to fill and thus takes less time to solve, but this consideration is insufficient for ranking Sudoku instances most of the time, so a new efficient technique must be implemented to have more accurate ranks.

3. Data set

To design the new Sudoku ranking technique, it is essential to first analyze real data in order to have a complete picture of all possible Sudoku scenarios. So, we tokenize previous study data, which includes 45 Sudoku instances of all rank levels and their solutions with different techniques [Mantere, 2007].

The following table summarizes the most important output we need from the historical data, which is the average time consumed in 100 runs for each instance from all difficulty levels with three different instances for each one of these levels; note that these instances and all data related to them are available on the library website [Sudoku page n.d].

Difficulty rating	Average Solving Time		
	a	b	c
1	0.055	0.026	0.179
2	0.308	1.058	0.424
3	0.606	2.109	1.292
4	1.632	2.814	2.168
5	2.759	2.01	3.988
E	0.022	0.011	0.021
C	0.762	2.348	1.691
D	7.616	3.489	2.135
SD	12.441	7.053	5.98
Easy	0.428	0.094	0.109
Medium	2.28	7.874	2.164
Hard	34.015	3.436	17.952
GA-E	0.158	0.196	0.151
GA-M	1.031	0.842	1.162
GA-H	6.405	3.693	24.441
Average | Sum	3.81	171.428	

Figure 2: historical data

We focused on this table because our objective in this work is to build a ranking technique strongly related to the time required to solve it.

4. Parameters extraction

Now, we are looking for reasonable parameters to be considered in the new ranking technique.

In order to understand why these instances vary in their ranks and take time, we analyzed them and inducted three parameters for each instance: the number of filled cells in the instance (InputFN), the minimum number of filled cells in rows and columns (MinRC), and the number of empty sub-squares (3*3) in the instance (EmptySQRS).

After defining the variables we expected to affect the difficulty of the puzzle, we computed the correlation between them using the SPSS statistic tools, and the following table results:

Correlations

		Time	EmptySQRS	InputFN	MinRC
Time	Pearson Correlation	1	.497	-.609*	-.535*
	Sig. (2-tailed)		.059	.016	.040
	N	15	15	15	15
EmptySQRS	Pearson Correlation	.497	1	-.208	-.411
	Sig. (2-tailed)	.059		.456	.128
	N	15	15	15	15
InputFN	Pearson Correlation	-.609*	-.208	1	.517*
	Sig. (2-tailed)	.016	.456		.048
	N	15	15	15	15
MinRC	Pearson Correlation	-.535*	-.411	.517*	1
	Sig. (2-tailed)	.040	.128	.048	
	N	15	15	15	15

*. Correlation is significant at the 0.05 level (2-tailed).

As the above table illustrates, the Time Pearson correlation row reasons the following main points;

- Correlation between the average time consumed in solving the instances and the number of filled cells is -.609, which means there is a strong negative correlation between time; in other words, when the number of filled cells increases, the time consumed in solving the puzzle will be decreased.
- Correlation between the average time consumed in solving the instances and the number of empty sub-squares 3*3 in the instances is -0.497, which means there is a positive correlation between time; in other words, when the number of empty sub-squares increases the time consumed in solving the puzzle will be increased also.
- Correlation between the average time consumed in solving the instances and the minimum number of filled cells in the instance's rows and columns -.535 which means there is a strong negative correlation between time.

After analyzing the correlation between the variables, dimensionality reduction has to be performed on these variables to ensure that no unnecessary variable was included in the study using PCA (Principle component analysis), a popular technique for analyzing datasets with a large number of dimensions/features per observation, increasing data interpretability while preserving the maximum amount of information, and enabling multidimensional data visualization. PCA is a statistical technique used to reduce the dimensionality of a dataset. The result is in the following tables:

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.771	59.036	59.036	1.771	59.036	59.036
2	.798	26.612	85.648			
3	.431	14.352	100.000			

Extraction Method: Principal Component Analysis.

Communalities

	Initial	Extraction
InputFN	1.000	.578
MinRC	1.000	.749
EmptySQRS	1.000	.444

Extraction Method: Principal Component Analysis.

The PCA first table shows the new variable, which is combined with the input variables, which causes a variance in data; its combination is illustrated in the second table, which shows that all three variables are enrolled in the new variable PCA, which pushed us to accept all of this variable as inputs to our following ranking technique.

5. Fuzzy logic rating

Many situations in logic could be adjudged as true or false, but some situations became complicated and required more detailed reasoning. Fuzzy logic is the solution in this domain; it provides valuable reasoning flexibility. Also, it considers the inaccuracies and uncertainties of any situation.

In general, fuzzy logic is based on the idea that the concept of true or false is too restrictive in many cases and that there are many shades of gray in between. It supports partial truths, which means that a statement can be partially true or false rather than completely true or false.

A pass/fail evaluation, for example, is, in most cases, insufficient to assess a student's academic performance. It requires a 100 percent evaluation in order to provide an accurate description of the student's mastery of course concepts and academic skills and to compare them to what is adequate. This score must be as universal as possible so that we can compare students from various areas.

Similarly, Sudoku instances must be evaluated using as scientifically accurate techniques as possible in order to inform the public about how difficult it is. This will be done in this work with the Fuzzy Logic mathematical method for representing ambiguity and uncertainty in decision-making; it accepts partial truths and is used in a wide range of applications. It is based on the concept of membership functions and uses Fuzzy rules to implement it.

The membership function, which defines the degree of membership of a rank level to a certain set or category, is the fundamental concept of Fuzzy Logic. The membership function converts an input value to a membership degree between 1 and 5, for instance.

The input parameters for the fuzzy ranker are the accepted parameters from the section above:

- The number of filled cells in the instance (InputFN).
- The minimum number of filled cells in rows and columns (MinRC).
- The number of empty sub-squares (3*3) in the instance (EmptySQRS).

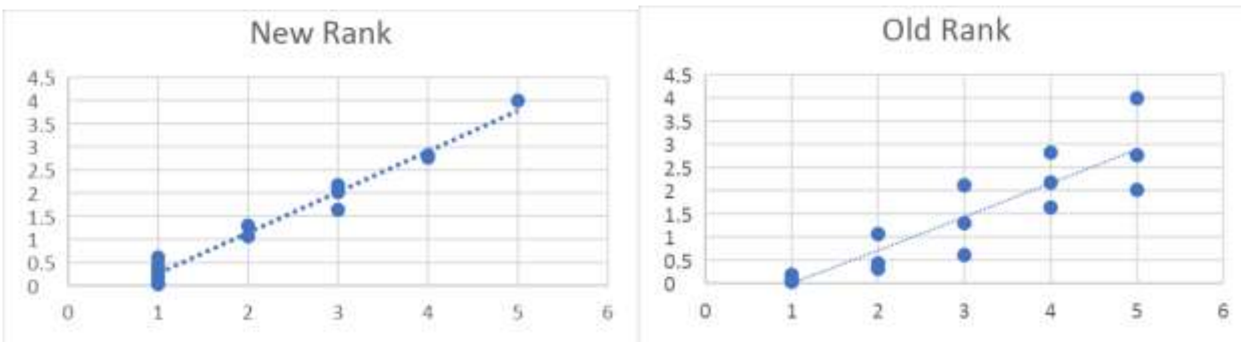
The output of the ranker ranks in the same category in the historical data, as shown in Figure 2. For example, the following table illustrates the output of the first category levels from 1 to 5:

	MinRC	EmptySQRS	InputFN	old rank	New Rank
s01a	2	0	33	1	1
s01b	2	0	36	1	1
s01c	1	0	32	1	1
s02a	2	0	30	2	1
s02b	0	0	28	2	2
s02c	2	0	28	2	1
s03a	2	0	28	3	1
s03b	0	0	26	3	3
s03c	1	0	27	3	2
s04a	0	1	28	4	3
s04b	1	0	27	4	4

s04c	0	2	28	4	3
s05a	1	1	30	5	4
s05b	0	0	28	5	3
s05c	1	1	26	5	5

6. Result and Discussion

As shown in the above table, the new rank differs from the old rank; to validate this new rank, we must present the relationship between it and the historical average time for these instances, using regression and r-square values, and the same method must be applied to the old rank to make a descriptive comparison.



The instance soling time is presented with its old rank in the right figure, and its relation with time could be computed with the regression equation time = 0.7273*rank - 0.7533, and the error could be calculated as $R^2 = 0.7878$. , and with the new rank output from fuzzy at the left side, its relation with time could be computed with the regression equation time = 0.8798*rank - 0.6243, and the error could be calculated as $R^2 = 0.9736$.

The new rank is related to time in a more linearity relation, with fewer sums of squares of residuals, which means the error percentage is very small, ensuring the validation of this new technique.

7. Conclusion

This paper proposes a new ranking technique for the Sudoku puzzle difficulty level that is based on fuzzy logic and takes into account three parameters: the number of filled cells in the instance, the minimum number of filled cells in rows and columns, and the number of empty sub-squares (3*3) in the instance. Those parameters are validated using correlation and dimensionality reduction techniques, and the output of the proposed fuzzy rating is compared with historical ratings, which shows that the new rank presents the puzzle difficulty more accurately.

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