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| RESEARCH ARTICLE

A Comprehensive Optimization Approach on Financial Resource Allocation in Scale-Ups

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| ABSTRACT
Many startups try to pass the transition phase and begin the scale-up phase successfully. However, few are able to survive during this phase. One of the most important factors that can assist these startups in the scale-up phase is managing their financial resource. By doing so, the startups can reduce the consumption of these resources and, at the same time, increase their productivity. Cash flow is considered the pillar of the financial resources in the transition phase, and by managing the cash flow consumption, the probability of surviving in the transition phase will increase. This study aims to propose a model for the startup's transition to allocate the optimal cash flow at the beginning of the scale-up phase. The components of the proposed optimization model are constructed based on the Mean-Variance framework, which was established by Harry Markowitz in 1952, to find the best composition of the cash flow allocation at each stage (financial period t). According to the cash flow statement, cash flows are separated into three categories: investing, operations, and financing activities. Finally, the model's mechanism is boosted by adopting the principles of the behavioral theory of the firm to form a reinforcement learning model for resource allocation at the edge of the transition/scale-up phase. Therefore, by utilizing the proposed model during the transition phase, entrepreneurs may plan for a successful scale-up before wasting financial resources to reach sustainable growth. This paper introduces a model that offers critical insights and a novel framework, paving the way for future research in this emerging area; the model serves as a significant foundation, highlighting key opportunities and setting a new direction for impactful advancements in the field.

| KEYWORDS
Cash Flow Allocation, Scale-Up process, Resource Allocation, Transition Phase, Mean-Variance Framework, Behavioral Theory of the Firm.

| ARTICLE INFORMATION
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1. Introduction
The OECD (Organization for Economic Cooperation and Development) defines a scale-up company as an enterprise with at least 10 employees and an average annualized return of at least 20% over consecutive three years (OECD, 2007). Transition is the phase between start-up and scale-up; when startups decide to sustain their return and become scale-up, they should first pass the transition phase. However, many of these startups are not able to survive (CB Insights, 2024). The performance of the scale-ups is not sustainable during the transition period. Some of these fluctuations are caused by changes in cash flow consumption, which affects the firm's financial resources. Although other factors influence the return's fluctuation, this study focuses only on the fluctuation due to the cash flow allocation. According to the cash flow statement, there are three categories of cash flow types for allocating in organizations: cash flow allocated to investing, operations, and financing activities. Because it is necessary to have a rich financial resource for scaling up, startups need different strategies for allocating their resources. According to the literature, a

unique cash flow prediction model should be developed in order to determine the optimal solution for distributing cash flow inside businesses (Navon, 2006).

The current study suggests a model for finding the best composition of the three types of cash flow that will be allocated in the transition phase. The optimization model is inspired by the Markowitz mean-variance portfolio theory, and by adopting the behavioral theory of the firm's decision-making process concepts, the model has been boosted to the reinforcement learning model. The transition phase is separated into the n financial periods; the length of each financial period is t. So, the startups will begin the transition phase and optimize the return by finding the best combination of the three different types of cash flows for allocating in investing, operations, and financing activities at the first stage, which is period t. Then, it will continue the process and go to the next stage, which is the financial period $t + 1$, and optimize the return by finding the best composition; then the model will compare this return with three parameters that are the expected return of the firm, the maximum historical return of the firm, and the average return of the market. If the optimized return is greater than the other three parameters, the startup will keep the combination; otherwise, if the optimized return is less than any of the above parameters, the firm should change the combination.

Therefore, when the optimized returns on the combination don't change after several equal financial periods t, and the optimized return always stands over three specified parameters in the same period, the model sets that combination of cash flow's allocation ratio as the optimized solution for allocating cash flow at the end of the transition phase. The findings of this research will help startups in the transition phase, where there is no specific information on how they allocate their cash flow for different activities. Accordingly, by utilizing this model, they will be able to gain the highest return based on their cash flow allocation.

In chapter two, the literature review is discussed, and comprehensive definitions of scale-ups, the transition phase, cash flow allocation, mean-variance portfolio theory, and the behavioral theory of the firm are elaborated. The next chapter consists of the methodology of the research. After completing the methodology, the results, discussion, and conclusion are presented in chapters 4 and 5, respectively.

2. Literature Review

2.1. Scale-up

Scale-ups are businesses having at least 10 workers at the start of the monitoring period and averaging a yearly increase in employees (or turnover) of more than 20% annually during three consecutive years (Cardenete & García-Tapial, 2018). The first phase of the company lifecycle is a startup; when the internal and external circumstances are favorable, and the startup survives after an initial two to three years, the growth phase should begin. In this phase, which is sometimes known as scaling up, businesses must create solid sales and marketing strategies, create an organization by employing and managing a variety of individuals, and understand how to get key inputs like the correct sources of funding and providers (Zajko, 2017). A scale-up firm is distinguished by its emphasis on quick growth in terms of turnover, innovation advancement, and future expansion into overseas markets (Zajko, 2017). According to Henrekson and Johansson's (2010) study, scale-up occurs when a small number of quickly expanding companies create an abnormally significant fraction of all new net jobs, particularly compared to non-high-growth companies (Henrekson & Johansson, 2010; Sanchez, 2020). Many scaling businesses show hyper-growth rates of more than 40% annually (Piaskowska et al., 2021). While it is essential to create capacity rapidly to support an increase in revenues and employment quickly (Autio, 2016; Sullivan, 2016), businesses face challenges managing successfully as they begin to expand (Desantola & Gulati, 2017; Sirmon et al., 2010), cope with internal conflict and a feeling of turmoil (Desantola & Gulati, 2017), and seize emerging opportunities to stimulate growth and start providing for even more economies of scale (Reuber et al., 2021) Scaling enterprises evolve fast in order to reach a competitive scale. The rate of expansion sometimes exceeds that of any other phase of the business life cycle (Coad, 2018). The need to manage the structural limits before the scale-up phase has been given significant attention in the literature. Scalability is frequently connected with a company's capacity to develop quickly without being inhibited by structural limits (Monteiro, 2019). The essential question is the magnitude at which a company can scale successfully and hence become a scale-up company (Coutu, 2014). Despite the vast volume of start-ups, relatively little has proven to scale up successfully (Rengers, 2022).

An organization's capacity to scale is the outcome of a successful transition phase. Sustainable growth and market dominance are ensured by a core business idea and a viable company strategy (Picken, 2017). The product-market fit is successfully established, and technical obstacles are removed. With sales and more resources to support this growth, the enterprise will grow quickly (Jirásek & Bílek, 2018; Miller & Friesen, 1984). The scale-up phase lasts until the company's growth rates match those of the market, at which point it becomes mature (Fisher et al., 2015). Scale-ups make significant benefits to the economy (Rengers, 2022).

2.2. Transition

The transition phase was initially included in the lifecycle model of an organization (Picken, 2017; Rengers, 2022). For the establishment of a mature business and for the success of the company, the transition phase is the most important phase (Picken, 2017). When a startup has identified and validated its business model and is ready to scale, the transition stage begins. As shown in Figure 1, it is the phase that works as a link between startups and scale-ups (Nadali et al., 2019).

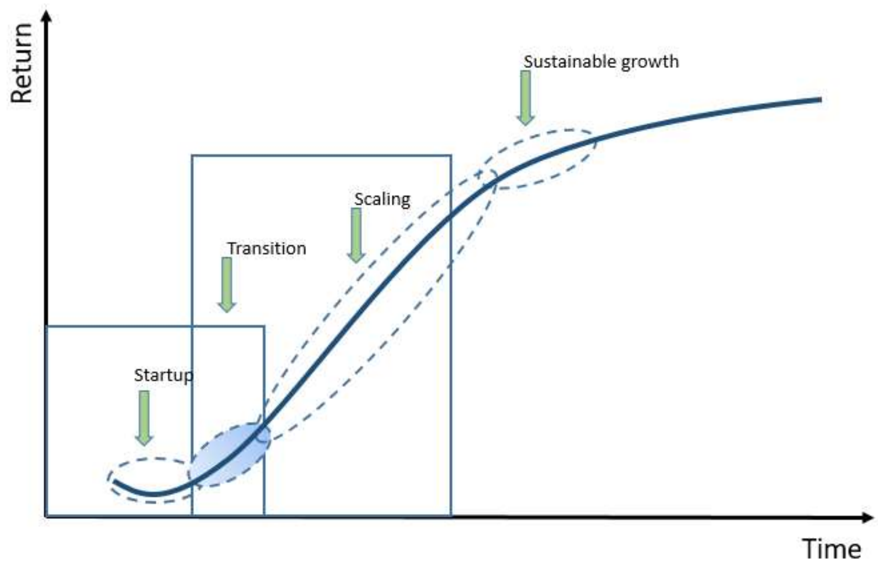

Figure 1. Transition phase, Picken, 2017, modified by authors

The transition process involves a lot of organizational changes (Flamholtz & Randle, 2007; Rengers, 2022). To sustain organizational growth, a start-up must implement a well-defined plan (Flamholtz & Randle, 2007). Joseph C. Picken (2017) defined eight transitional challenges that were used to create risk categories. These eight best practices are: 1. Defining a goal and keeping focus; 2. Placing products/services in a larger market; 3. Maintaining customer/market responsiveness; 4. Building an organization and management team; 5. Building effective procedures and infrastructures; 6. Building financial ability; 7. Developing a suitable culture; 8. Managing risks and weaknesses (Nadali et al., 2019). Many of the aforementioned challenges will be solved by managing resource allocation and, specifically, cash flow allocation.

2.3. The relation between resource allocation and scaling up:

The four key areas of decision-making in the process of a firm scaling up are Strategy, People, Execution, and Cash (Harnish, 2014; Zajko, 2017). According to research on 3,200 start-ups, Marmer et al. (2011) created a systematic approach for measuring the readiness of businesses to scale up. Business Model, Customer, Team, Product, and Financials were the five interdependent key aspects they used to analyze each start-up. They emphasize that having these five main aspects in equilibrium will help a start-up that wants to become a high-growth firm develop at the highest possible rate (Zajko, 2017). According to Marmer's study, one of the main factors of successful scale-up is managing financial resources. In addition, Strategy changes, human resource changes, administrative changes, cultural changes, and leadership changes were all grouped together as organizational changes through the transition phase (Chen et al., 2018; Luo & Jiang, 2014; Rengers, 2022; Wit & Meyer, 2010). Many of these changes are related to utilizing the resources in the organization.

The Penrose theory (Penrose, 1995), which is a dominant theoretical approach to explaining firm growth (Nason & Wiklund, 2015), highlights the allocation of resources for "productive services" that provide growth. Productive services are growth-enabling activities in the context of scaling companies. Given the limitation of resources when expanding companies' progress beyond the start-up period, resource acquisition and development is important (Piaskowska et al., 2021). Scale-up companies will develop a specific pattern in their growth-enabling operations over time as they develop them.

According to (Harnish, 2014), scaling up has three challenges: 1. Scalable infrastructure: the lack of physical and organizational systems and structures to control the growing complexity in communication and decision-making that comes with growth. 2. Leadership: the organization's inability to hire or develop enough leaders throughout the organization who have the potential to delegate and predict.3. Market dynamics: failing to manage the increased competitive pressures that emerge (and reduce profitability) as your organization grows. Scaling-up businesses should focus on the four key aspects of decision-making: managing Cash, leading People, defining Strategy, and driving Implementation. As a result, the firms should be experts in the above-mentioned principles.

Due to Flamholtz and Kurlan's (2005) research, there are six main tasks for organizational growth and development: identification and definition of a valid market niche; development of goods or services for the chosen market niche; acquisition and development of resources needed to operate the organization; development of day-to-day operational systems; development of management systems required for the long-term operating of the company; development of the organizational cues (Flamholtz et al., 2005).

2.4. The importance of managing cash flow in the scale-up phase

Cash flow forecasting is a continuous activity, just like other management tools. When a project is ongoing, performance is evaluated and compared to the projections, which provide the project's forecast or target (Navon, 2006). Because cash is the most significant corporate resource for a company's daily operations, having an effective cash-flow management system is critical for survival (Navon, 2006; Peer, 1982). Acquiring and gaining access to funds is referred to as financing activity, which is critical for overall business growth (Nason & Wiklund, 2015) and scaling (Duruflé et al., 2017). Financing activity refers to the acquisition and use of funds.

Financing activities may facilitate scale-up companies to have access to external abilities in the form of expertise from their investors. For instance, businesses that have raised venture capital frequently profit from having access to their investors' experience, strategic recommendations, coaching, and venture capabilities (Meglio et al., 2017). It is extremely important for growing businesses since they may use tacit knowledge to accelerate their learning (Monaghan & Tippmann, 2018). Furthermore, venture capital financing behaves as a crucial indicator of potential growth, and companies benefit from venture capitalists' reputation and prestige to increase their legitimacy (Davila et al., 2003; Fernhaber & Mcdougall-Covin, 2009). These arguments demonstrate that financing activity helps scale-up firms to obtain a critical versatile resource, therefore encouraging a specific growth-enabling activity (Piaskowska et al., 2021). Another strategy that could help scale-up organizations to be successful in the transition phase would be decreasing the consumption of resources with higher returns.

2.5. Metrics for scale-up:

According to Barbero Navarro et al. (2012), high-growth SMEs have a yearly growth rate of more than 10% over a 5-year period. Accordingly, Sandberg & Hofer (1987) find that a very successful organization has achieved profitability, and equity investors have earned profits of at least 30% per year when disposing of shares. According to Ács (2015), high-impact businesses are younger and smaller than other businesses, hiring between 1 and 19 people—and the majority are not new entrepreneurs (Sanchez, 2020). According to Coutu's (2014) research, SMEs with 10 or more workers and sales growth of 20% per year over a three-year period should be considered "scale-ups." According to NESTA (2013), a scale-up is an enterprise that has at least 10 workers at the beginning of the period and has an annual average increase of 20% in employment over a period of three years. The types of performance metrics for scaling up entrepreneurship are summarized in Table 1.

Table 1. Performance metrics of scale-up (Sanchez, 2020)

Objective measures	Authors
Sales have more than doubled in the last four years, with an employment growth quantifier of two or larger.	**(Ács, 2015)**
The majority of "high impact firms" (between 1 and 19 persons) are not start-ups, and they are smaller and younger than other firms.	
SMEs with an annualized return of more than 10% over a five-year timeframe in terms of sales.	**(Barbero Navarro et al., 2012)**
Extreme growth for three to four years, SMEs consistently double and redouble their revenues.	**(Chetty & Campbell-Hunt, 2003)**

Objective measures	Authors
Organizations with rapid expansion raise sales by at least 50% over three consecutive financial years.	**(Autio, 2016)**
Rapid-growth businesses are defined as those with a 3-year steady annual revenue growth of 80% or more.	**(Barringer et al., 2005)**
Revenue of currently growing enterprises increases by more than 25% yearly over a short period of time.	**(Roure & Keeley, 1990)**
Over the first seven years, a firm experiencing rapid development "doubles return in real-time."	**(Littunen & Tohmo, 2003)**
For a scale-up business to be established, four factors are crucial: The company must meet the following requirements: (1) have at least five years of sales history; (2) first-year revenues of no more than $25 million; (3) no sales drop in years four and five; and (4) be publicly traded and standalone in year five.	**(Hambrick & Crozier, 1985)**
Three years compound annual growth rate of 40% or greater" is the definition of a rapid growth company (based on yearly sales). In the same time frame, a slow-growing company has a "compound annual growth rate of 5% or below.	**(Zhang et al., 2008)**
In four years, a company with one job must increase by 500 percent. A company with five or more jobs is anticipated to see 100 percent growth within the same time period.	**(Duruflé et al., 2017)**
Businesses with more than 150 percent growth are referred to as "hyper-growth," those with 50 percent to 150 percent growth as "strong growth," those with less than 50 percent growth as "slow growth," and those with less than 50 percent growth as "declining firms."	**(Halabisky et al., 2012))**
During the first three years and the first five years after their founding, high-growth firms see "average annualized growth of more than 20% per year."	**(OECD, 2007)**
"Scale-ups are not start-ups" and should comprise SMEs with 50 or more workers, increasing sales by 20% per year over a three-year period.	**(Coutu, 2014)**
Entrepreneurs that have "experienced predicted revenue growth (incomes) of 20 percent or more each year," have been in business for more than 10 years, and employ more than thirty-three individuals are considered high-growth entrepreneurs.	**(Morris, 2011)**

2.6. Cash flow forecasting

Chandler (1962) characterized business strategy as the establishment of goals and objectives, as well as the resource allocation required to support these aims. Ansoff (1965) stressed the importance of resource allocation in strategic planning and the requirements for a resource budget. A robust cash flow management system is vital for obtaining loans since banks and other lending organizations are far more likely to give money to firms that can provide monthly cash flow predictions (Navon, 2006).

Usually, there are differences between the projected and actual output and the plan must be altered in order to fulfill the initial objective or be as similar to it as possible (Navon, 2006). There are several methods for projecting cash flows, each with a different level of accuracy and precision. Due to capacity restrictions, not all cash-flow forecasting methods can be described in this study. The initial approach is manual integration, as suggested by Sears (1981). The second method is founded on a schedule and a bill of quantities (BOQ), proposed by Booth et al. (1991) and Mawdesley et al. (1989). Other strategies employ analytical and simulation techniques. Bennett and Ormerod (2006) created a cash-flow prediction simulation model known as the hierarchy of bar charts (Navon, 2006).

Reinschmidt and Frank (1976) established a model for pre-estimating cash flow prediction. The model is supplied with 100-200 actions, together with their associated costs and linkages. A simulation model is utilized to assign random lengths to the activities. Then, an initial network, as well as statistically dispersed cash flow and interest payments, are created. The model accepts monthly or annual cash flow limits and reorganizes the schedule accordingly. However, a novel cash flow prediction model should be constructed in order to find the best solution for allocating cash flow inside the organizations.

2.7. Mean-Variance Portfolio Theory

2.7.1. Elements of decision problems under uncertainty

A set of alternatives and their corresponding probabilities of happening, known as a frequency function or return distribution, must be used to represent the payment. The main model looked at the two most commonly used characteristics of such a distribution: a measure of central tendency known as the expected return and a measure of risk or deviation from the mean known as the standard deviation (Harry Markowitz 1952). The risk on a portfolio is more complicated than the risk on single assets, which is a key finding of the main research, so the theory emphasizes that traders should not keep individual assets; instead, they maintain sets or portfolios of assets. It relies on whether the returns on distinct assets prefer to move jointly or if some assets provide strong returns while others provide poor returns.

Initially, the mean-variance framework analyses only two asset portfolios. The model gave a comprehensive geometric and algebraic study of the features of two asset portfolios with varying estimations of how they covary together. This methodology is then extended to the situation of several assets. Finally, it arrives at the opportunity set that the investor faces in a risky environment. Using the newly introduced summation notation and a bar above a variable to represent the expected return, the main model of mean-variance obtains the expected value of the M equally likely returns for the asset i as follows, R_{ij} represents the Jth possible outcomes of the return for the asset i in the portfolio (Elton et al., 2009).

$$\bar{\boldsymbol{R}} = \sum_{j=1}^{M} \frac{\boldsymbol{R_{ij}}}{\boldsymbol{M}}$$

In the mean-variance framework, the expected return on the asset is illustrated both by E(R_i) and $\bar{R}$. The following principles are useful for further computations:

First principle: The expected value of the sum of two returns is equal to the sum of the expected value of each asset's return:

$$E(\boldsymbol{R_{1j}} + \boldsymbol{R_{2j}}) = \bar{\boldsymbol{R}}_1 + \bar{\boldsymbol{R}}_2$$

Second principle: Constant times the expected return is equal to the expected return of the constant times return of the asset:

$$\boldsymbol{E}[(\boldsymbol{C}\boldsymbol{R_{1j}})] = C\bar{\boldsymbol{R}}_1$$

2.7.2. A measure of dispersion

It is essential to have some metrics of how far the results vary from the average in addition to finding a value of the average return. In the main model, the deviation of the return is computed by deducting the outcomes from the average, and after computing this amount for each asset, the overall measure can be obtained by taking an average of this difference.

Although this seems reasonable on the surface, there is a problem. Positive and negative distinctions will exist, but they will mostly balance one another, which leads to the difference in average for an item with a highly variable return that might not need to be as significant as the average difference for an asset with a high solid return. The average value of this difference must actually always be exactly zero. To solve this problem, the model has squared all differences before determining the average (Elton et al., 2009).

$$\sigma_i^2 = \sum_{j=1}^{M} \frac{(R_{ij} - \bar{R}_i)^2}{M}$$

The expected return on the assets is evaluated from the past trend of the assets, and the risk of a portfolio of assets differs significantly from the risk of single assets. Most notably, the variation of a combination of two assets may be smaller than the variance of either item alone.

2.7.3. Characteristics of portfolios in general

According to (Elton et al., 2009), the simple weighted average of the returns on each asset makes up the return on a portfolio of assets. Each return is given a weight based on how much of the portfolio is allocated to that item. If N is the number of assets, X_i is the percentage of the individual's money invested in the ith asset and $R_{pj}$is the jth return on a portfolio, then

$$R_{pj} = \sum_{i=1}^{N} (X_i R_{ij})$$

Additionally, the expected return is a weighted average of the expected returns on the various assets. Applying the expected value of the previously stated equation for the return on a portfolio results in

$\bar{R}_p = E(R_p) = E(\sum_{i=1}^{N} X_i R_{ij})$

Which is equal to: $\bar{R}_p = \sum_{i=1}^{N} E(X_i R_{ij})$

That could be written as: $\bar{R}_p = \sum_{i=1}^{N} (X_i \bar{R}_i)$

The variance served as the second moment. Simply put, a portfolio's variance, denoted by the symbol. $\sigma^2 p$, is the expected value of the squared deviations between the portfolio's return and its mean return or $\sigma^2 p = E(R_p - \bar{R}_p)^2$. The variance of the portfolio with two assets based on the main model would be :

$$\sigma_p^2 = E\left[X_1^2(R_{1j} - \bar{R}_1)^2 + 2X_1X_2(R_{1j} - \bar{R}_1)(R_{2j} - \bar{R}_2) + X_2^2(R_{2j} - \bar{R}_2)^2\right]$$

By applying the first and second principles, the equation becomes:

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_{12}$$

As previously proven, when two assets have positive and negative outcomes at the same time, acquiring a portfolio consisting of two assets does not lower risk. The variance formula for a portfolio may be extended to include more than two assets. Imagine a three-asset situation first. Substituting the expression for portfolio return and expected return into the basic calculation for variance becomes :

$$\sigma_p^2 = E\Big[X_1^2(R_{1j} - \bar{R}_1)^2 + X_2^2(R_{2j} - \bar{R}_2)^2 + X_3^2(R_{3j} - \bar{R}_3)^2 + 2X_1X_2(R_{1j} - \bar{R}_1)(R_{2j} - \bar{R}_2) + 2X_1X_3(R_{1j} - \bar{R}_1)(R_{3j} - \bar{R}_3) + 2X_2X_3(R_{2j} - \bar{R}_2)(R_{3j} - \bar{R}_3)\Big]$$

Considering $\sigma_{12=}\sigma_{21}$ *by* applying the first and second principles, the equation could be written as follows (Elton et al., 2009).

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + 2X_2X_3\sigma_{23}$$

The formula can be extended when the number of assets in the portfolio is more; the general formula would be:

$$\sigma_p^2 = \sum_{j=1}^{N}(X_j^2\sigma_j^2) + \sum_{j=1}^{N}\sum_{\substack{K=1\\K\neq j}}^{N}(X_jX_K\sigma_{jk})$$

This study focuses on the portfolio formula with 3 assets, so the research will not discuss the portfolio with N assets.

2.8. The behavioral theory of the firm

Simon (1947) puts out a theory of human decision-making that seeks to take into account both the rational features of choice, which have been the main focus of economists, and the characteristics and constraints of human decision-making processes, which have drawn the concern of psychologists. Simon largely emphasizes internal organizational decision-making processes, describing how organizations affect their employees' decisions, ensuring consistency between those decisions, and ensuring that the decisions are congruent with the broader corporate goals (Cyert & March, 1992). Cyert and March (1963) place a strong emphasis on the actual process of making business decisions and offer in-depth analyses of the methods used by organizations to carry out this process. A general theory of economic decision-making by a commercial enterprise that is process-oriented and empirically relevant was developed by Cyert and March (1963). The foundations of a behavioral theory of the firm are presented by Cyert and March (1963), and they have been shown to be pertinent to both economic theory and the theory of complex organizations (R. Cyert & March 1963). Cyert and March (1963) place a strong emphasis on the actual process of making business decisions and offer in-depth analyses of the methods used by companies to carry out this process. (R. Cyert & March, 1963). Where Simon's Administrative Behavior (Simon, 1947). left off, Simon's Models of Bounded Rationality continue the effort to recognize decision-making in its broadest sense and, in particular, to demonstrate how economics and psychology can help shed light on business decision-making mechanisms (Cyert & March 1963).

A comprehensive strategy and timetable do not predetermine behavior in the organization once and for all. The routine has the nature of a dynamic capacity rather than a fixed procedure despite the fact that it is extensively routine. According to the figure. 2, they offer their broad framework for intra-organizational choices in March and Simon (1958, p. 48). The following are the key phases in March and Simon's behavioral model (March et al., 1993).

1. The lower the satisfaction of the individual, the more search for alternative programs the individual will undertake.
2. The more search, the higher the expected value of the reward.
3. The higher the expected value of the reward, the higher the satisfaction.
4. The higher the expected value of the reward, the higher the level of aspiration of the individual.
5. The higher the level of aspiration, the lower the satisfaction.

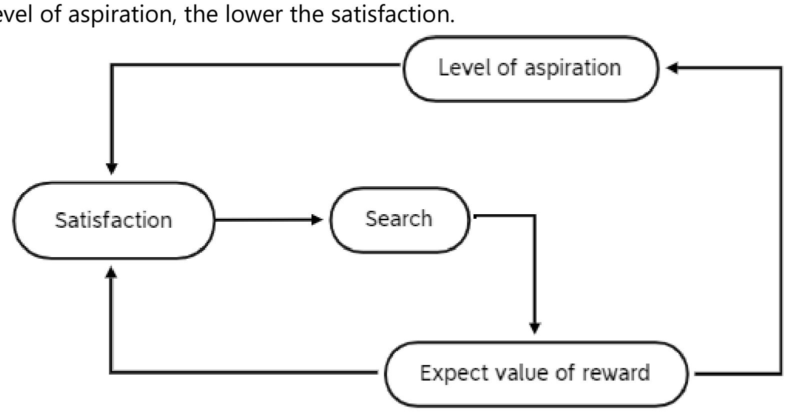

Figure 2. General model of the adaptive-motivated model (Cyert & March 1963)

3. Methodology

As mentioned in Chapter Two, the mean-variance framework aims to maximize the return of the portfolio with the lowest risk on the total portfolio. By utilizing the mean-variance framework concepts, current research aims to find a model for resource allocation in the transition phase of the firm lifecycle so the startups can find a way to pass this phase successfully and become scale-up. The following model used the perception of Dobrovolskiene and Tamošiuniene (2016) for utilizing the mean-variance framework.

We only focus on the financial resources in this research. As cash flow is the best index for evaluating financial resources, this study is conducted to optimize the return at the edge of the transition/ scale-up phase with minimum cash flow allocation. After optimizing the return at each specific period (for instance, every three months), the actual return will be compared to the market return and the maximum historical return of the startup. The model will regulate the optimized return based on the expected return by adopting the behavioral theory of the firm, so the model will boost and become a reinforcement learning model that automatically decreases the deviation of the expected return and actual optimal return. The deviation of the actual return from each of the aforementioned returns will multiply by a large number. After multiplying the large number by the difference in the returns, the model will eliminate the large outcomes. So, after the editing process, only those outcomes will remain, which will have negligible differences from the expectation, market, and maximum historical return.

Due to the cash flow statement, cash flows are separated into three different categories: cash flows for investing, operations, and financing activities. Therefore, instead of considering assets in the Mean-Variance framework, we considered these three types of cash flows in our model.

So j_1, j_2, j_3 will be defined as the following:

$$j = \begin{cases} \textbf{1. Investment} \\ \textbf{2. Financial} \\ \textbf{3. Operational} \end{cases}$$

The model's aim is to minimize the consumption of the cash flow and maximize the return at the same time. So, we consider the cash flow consumption instead of the variance in the portfolio. The startup in the transition phase may use a different combination of these three cash flows, which influences the performance of the firm in this phase. Therefore, changing this combination would be one of the main reasons that a startup's performance fluctuates in the transition phase.

$$\boldsymbol{\sigma^2 = CF^2}\ ,\ \boldsymbol{t = Financial\ Period},\ \boldsymbol{n = Whole\ transition\ period}$$

The objective function is the return function in this model, and the constraint function is the cash flow's consumption, which shouldn't exceed the specific fixed amount in each financial period. In the objective function, CF_{jt} is the ratio of the cash flow consumption to the cash flow type j in the period t, and R_{jt} is the return on spending cash flow on cash flow type j in period t.

$$\boldsymbol{MAX\ R_t = \sum_{j=1}^{3}\sum_{t=1}^{n} CF_{jt}R_{jt} \rightarrow Objective\ Function}$$

Since the total amount of cash flow will be allocated to only these three types of cash flow, the sum of consumption of these three cash flows would be 100 percent in total.

$$\boldsymbol{\sum_{j=1}^{3}\ \sum_{t=1}^{n} CF_{jt} = 1}$$

In order to compute the return of the asset in the portfolio, the Nomeda and Rima model utilized the following property (Dobrovolskiene & Tamošiuniene, 2016), which is adopted for computing the return on the project j.

$$\boldsymbol{R_i = \frac{1}{n}\sum_{i=1}^{n}\frac{G_i - I_i}{I_i} \rightarrow For\ Projected\ j\ In\ Original\ Model\ of\ Nomeda\ and\ Rima}$$

$\boldsymbol{R_i}$: ***Average return on project i***

$\boldsymbol{G_i}$: ***Gain on the project i***

$\boldsymbol{I_i}$: ***invest on the project i***

$\boldsymbol{n}$: ***Number of the senarios***

Therefore, in this study, the property has changed to compute the return on the cash flow allocation in the period t on cash flow type j.

$$\boldsymbol{R_{tj} = \sum_{t=1}^{n}\sum_{j=1}^{3} \frac{G_{tj} - S_{tj}}{S_{tj}} \rightarrow Return\ on\ CF\ cunsumptions\ of\ cash\ flow\ type\ j\ For\ Period\ t}$$

$\boldsymbol{R_{tj}}$: ***Return of cash flow type j at the financial period t***

G_{tj}: $Gain\ on\ the\ cash\ flow\ cunsumption\ j\ at\ the\ financial\ period\ t$
S_{tj}: $Spending/cash\ flow\ cunsumption\ of\ cash\ flow\ type\ j\ in\ the\ period\ t$
t: $Financial\ period\ t$

After specifying the objective function, the constraint function should be discussed. The constraint function consists of two properties; the first component is the variance of each cash flow type from its expected return, which is multiplied by the square of its proportion of consumption. The second component is the covariance between the consumption of each of the two cash flow types. Total cash flow consumption in the financial period t should be less than the specific cash flow identified by the startups according to their cash flow constraint.

$$CF_t^2 = \sum_{j=1}^{3}(CF_j^2 CFV_j^2) + 2\sum_{j=1}^{3}\sum_{k=1}^{3}(CF_j CF_k CFV_{jk}) \leq Specific\ cash\ flow$$

$$CFV_j^2 = \frac{1}{3}\sum_{j=1}^{3}\sum_{t=1}^{n} (R_{jt} - ER_{jt})^2$$

CF_t^2: $Total\ chash\ flow\ consumption\ in\ financial\ period\ t$
CF_j^2: $square\ of\ the\ consumption's\ ratio\ of\ cash\ flow\ in\ each\ cash\ flow\ type\ j\ in\ the\ period\ t$
CFV_j^2: $Sum\ of\ the\ variance\ in\ each\ cash\ flow\ type\ in\ the\ period\ t$
CFV_{jk}: $Covariance\ of\ the\ cash\ flow\ type\ j\ and\ k\ in\ the\ period\ t$
R_{jt}: $Return\ on\ cash\ flow\ allocation\ of\ the\ cash\ flow\ type\ j\ in\ the\ period\ t$
ER_{jt}: $Expected\ return\ on\ cash\ flow\ allocation\ of\ the\ cash\ flow\ type\ j\ in\ the\ period\ t$

To calculate the covariance of the cash flow type i and j in the period t, the following properties should apply.

$$CFV_{ij} = \frac{1}{n-1}\sum_{i=1}^{3}\sum_{j=1}^{3}\left[R_i - ER_i\right]\left[R_j - ER_j\right],$$

R_i: $Return\ on\ the\ cash\ flow\ type\ i\ in\ the\ period\ t$
ER_i: $Expected\ return\ on\ the\ cash\ flow\ type\ i\ in\ the\ period\ t$
R_j: $Return\ on\ the\ cash\ flow\ type\ j\ in\ the\ period\ t$
ER_j: $Expected\ return\ on\ the\ cash\ flow\ type\ i\ in\ the\ period\ t$
n: $Total\ number\ of\ analysing\ financial\ period\ t$

In order to find the optimal combination of the cash flow allocation, we need to take the first and second-order derivatives of the objective function, which is subjected to the constraint function. After finding the local maximum return in each period t, by adopting the "Behavioral Theory of the Firm" decision process mechanism, we compare this return with the expected return of the startup, its maximum historical return, and the return of the market in the period t. In this way, we have shaped a reinforcement model. The decision-making process and the model process are illustrated in Figure 3 and Figure 4, respectively.

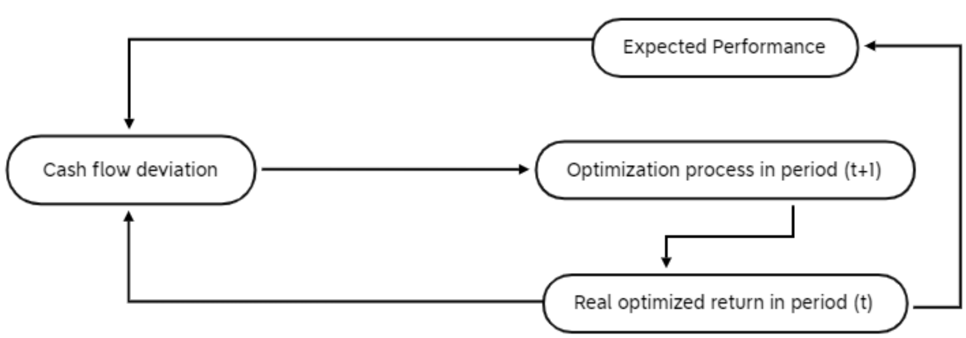

Figure 3. Cash flow adaptive-optimization model

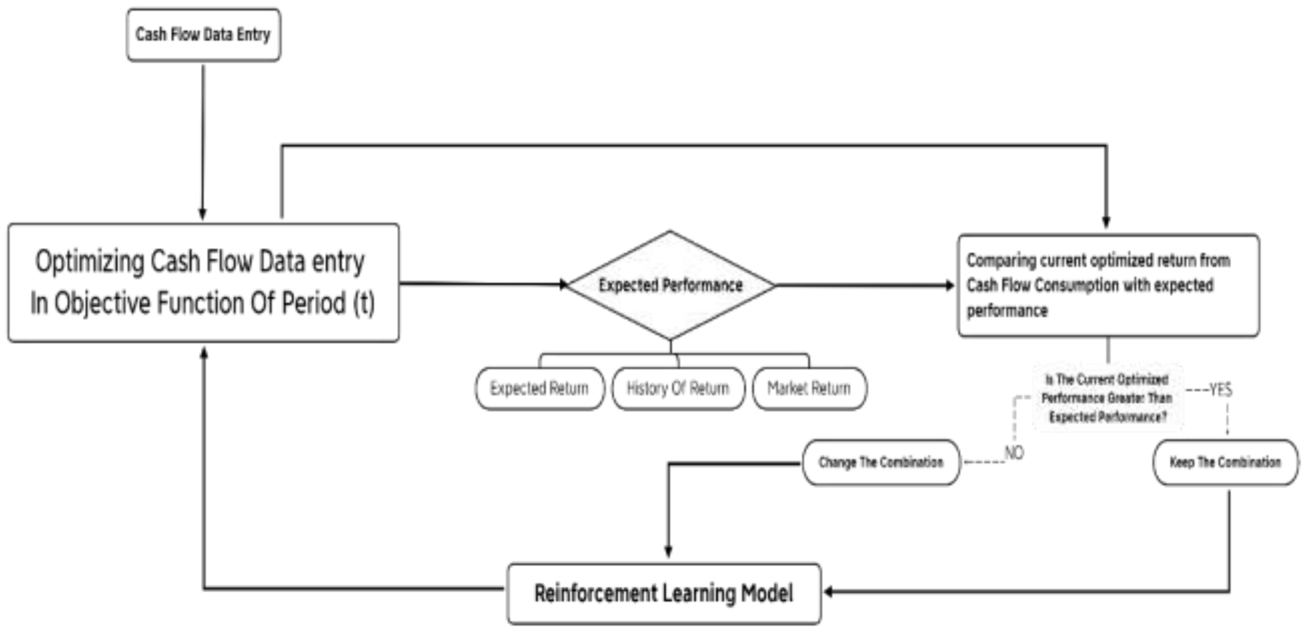

Figure 4. Conceptual framework of optimizing cash flow allocation in a transition phase

Therefore, to compare the maximized return with the market, maximum historical, and expected return, we added three more constraints to the model. The α, β, and γ are the specific amounts that show the threshold that maximized return can differ from the aforementioned returns and will be specified by the firms depending on how much they want to be precise.

$$\sum_{t=1}^{n}(R_t - ER_t)^2 < \alpha,\ \sum_{t=1}^{n}(R_t - R_{mt})^2 < \beta,\ \sum_{t=1}^{n}(R_t - R_{mht})^2 < \gamma$$

ER_t: *Expected return of the allocating cash flow in the period t*

R_{mt}: *Average return of the market in the period t*

R_{mht}: *Maximum historical return in the period t*

R_j: *Optimized return in the period t*

To calculate the total expected return of the portfolio, the following property should be calculated, which is the ratio of the allocation of each cash flow type i in period t times the expected return on each cash flow type in the period t.

$$ER_t = \sum_{i=1}^{3}\sum_{t=1}^{n} CF_{it}ER_{it},\ \sum_{i=1}^{3} CF_i = 1$$

This property is adapted from the original model of Dobrovolskiene and Tamošiuniene (2016). The original model utilized the following property to compute the expected return of the assets portfolio.

$$ER_p = \sum_{i=1}^{n} x_i E_i R_i \rightarrow \textit{In Original Model},\ \sum_{i=1}^{n} x_i = 1$$

4. Results and Discussion

As already mentioned in Chapter Two, when startups decide to transform to scale-up, they must pass the transition phase. During the transition phase, the performance of these startups will fluctuate. Although there are many reasons that cause this fluctuation, in this study, we only emphasize the fluctuation respected to the cash flow allocation. The change in return is due to utilizing different combinations of three types of cash flows which are cash flow allocation for investing, financing, and operational activities. The financial periods are divided into the n periods, and the length of each period has been shown by t.

Since at the edge of the startup/ transition phase, there is no financial Data about the transition phase; we need to find the optimum combination at each stage (which is period t) and then compare the optimum combination with other metrics. These metrics are market, maximum historical, and expected return of the firm. The model will reward the optimum combination if it is greater than the aforementioned returns and keep it in the system, but it will punish the combination and eliminate it if the return on the combination is less than the three sophisticated returns. That's why we consider three metrics to make the model a reinforcement learning model. Figure 5 illustrates the conceptual example of the optimum local maximums in each financial period t and compares the local optimum at each financial period with three lines.

The top line is the expected return of the firm in the period t, the middle line is the maximum historical return since starting the transition period, and the lowest line is the average return of the market in period t. The place of these lines may change in different financial periods. After finding the optimized return in the financial period t, if the return were higher or equal to all of the mentioned lines, the combination of consumption on cash flow would be acceptable at that point. In the case that even one of the mentioned lines is placed over the optimized return point, the combination will change for the next financial period. The startup will continue this process until it achieves the same maximum combination in several financial periods, during which, at all of them, the local maximum would be at the top of the three aforementioned lines.

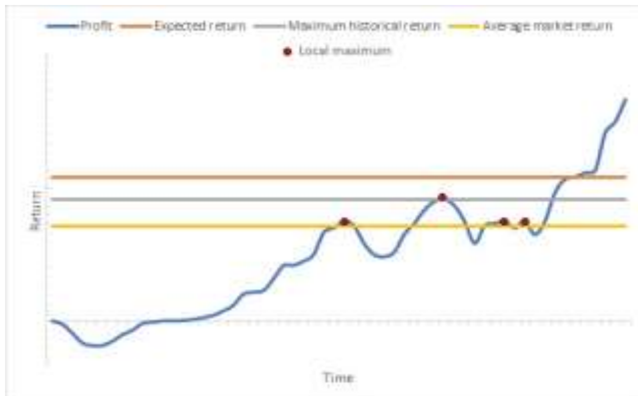

Figure 5. Boosting optimization model to reinforcement model

So, by utilizing this model, the startup will begin the scale-up phase with the optimum combinations of cash flow allocation instead of using traditional cash flow allocation.

5. Conclusion

As it has been mentioned the mean-variance framework can reduce the risk of the portfolio and make it close to zero, but there are existing risks in the market that cannot be eliminated due to the characteristics of the market. The situation is the same for our model. Although this model reduces the cash flow consumption for the firm at the end of the transition phase, there are some elements for implementing the model that affect the optimum combination and don't allow the waste of the cash flow to become zero. These elements are environmental, human, and infrastructural factors that cause unsuitable utilization of the model.

One of the most important factors that lead startups to fail in the transition phase is lacking financial resources during this period. The designed model of this study is capable of finding a way to reduce financial resource consumption and find the optimum solution for cash flow allocation in the transition phase. Therefore, allocating the cash flows to different startup activities with this model could prevent these enterprises from failing. The current study focuses only on cash flows as one of the main elements of financial resources and will optimize the return in a transition phase. The authors of this study suggest adopting the Mean-Variance framework in other processes and in different types of enterprises.

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